Coupled Adjoint-based Sensitivities Using the SU2 Native FSI Solver

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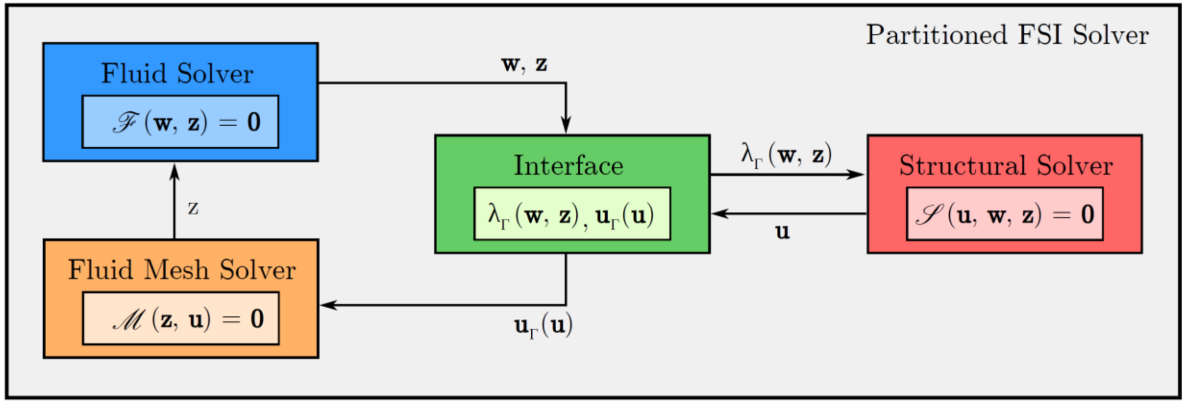
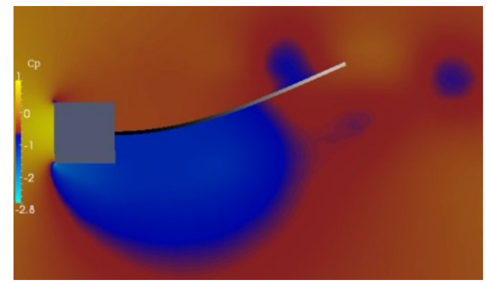
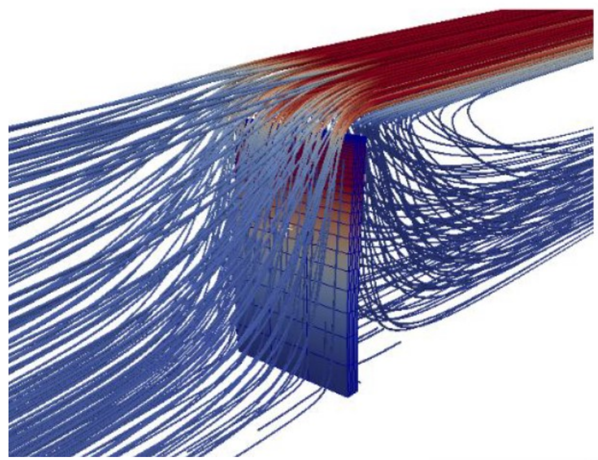
SU2 Developers’ Meeting, December 2017

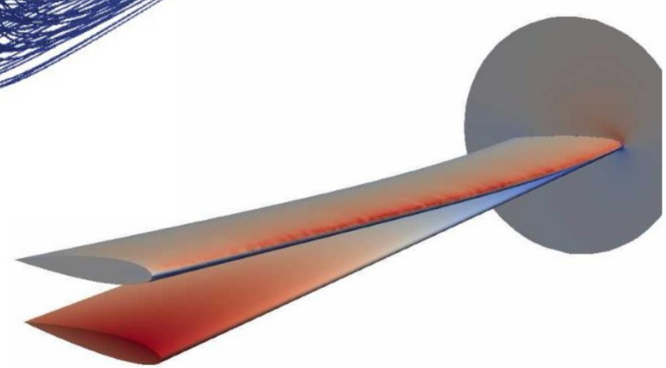
Current features

- Three-field solver
- Fully **implicit**
- **ALE** formulation
- “Elastic” mesh
- **Consistent** interpolation
- FEM from **finite-strain** solid mechanics
- Library of materials
- Static/dynamic
- Differentiated using **AD**
- **Parallel** implementation

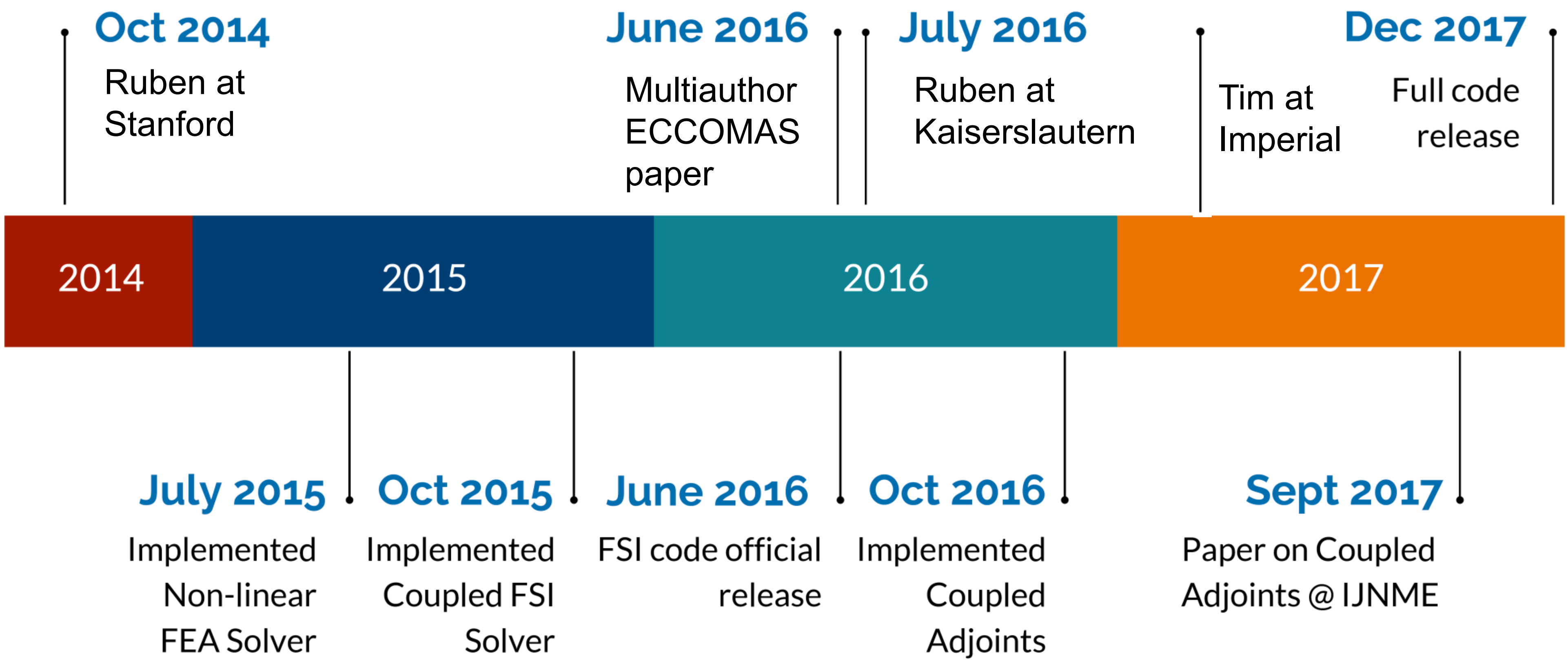








Timeline of the native FSI solver in SU2



*Ruben Sanchez (2017) *A Coupled Adjoint Method for Optimal Design in Fluid-Structure Interaction Problems with Large Displacements*, PhD thesis, Imperial College

Native vs. python

- Native FSI simulations from config file with multizone
  - Restricted to TETRA and HEXA
  - Isotropic materials (element-based definition)
  - Coupled adjoint
- Python wrapper by Terrapon’s group at U. Liege
  - Co-simulation with third-party FE solver
  - Interpolation and management via CUPyDO
  - Versatile

Evaluation of the coupled adjoint

1) Three-field problem as fixed-point

$$\min \quad J(\mathbf{u}, \mathbf{w}, \mathbf{z}, \boldsymbol{\alpha})$$

$$\text{subject to} \quad \mathbf{u} = \mathbf{S}(\mathbf{u}, \mathbf{w}, \mathbf{z}, \boldsymbol{\alpha})$$

$$\mathbf{w} = \mathbf{F}(\mathbf{w}, \mathbf{z}, \boldsymbol{\alpha})$$

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2) Lagrangian with co-states

$$\begin{aligned}\mathscr{L}(\mathbf{u}, \bar{\mathbf{u}}, \mathbf{w}, \bar{\mathbf{w}}, \mathbf{z}, \bar{\mathbf{z}}, \boldsymbol{\alpha}) = \\ & J(\mathbf{u}, \mathbf{w}, \mathbf{z}, \boldsymbol{\alpha}) \\ & + \bar{\mathbf{u}}^T \left[\mathbf{S}(\mathbf{u}, \mathbf{w}, \mathbf{z}, \boldsymbol{\alpha}) - \mathbf{u}\right] \\ & + \bar{\mathbf{w}}^T \left[\mathbf{F}(\mathbf{w}, \mathbf{z}, \boldsymbol{\alpha}) - \mathbf{w}\right] \\ & + \bar{\mathbf{z}}^T \left[\mathbf{M}(\mathbf{u}, \boldsymbol{\alpha}) - \mathbf{z}\right]\end{aligned}$$

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3) Solve the co-states

(In a second)

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3) Solve the co-states

(In a second)

4) Compute sensitivities

$$\frac{\mathrm{d}J}{\mathrm{d}\boldsymbol{\alpha}} = \frac{\partial J}{\partial \boldsymbol{\alpha}} + \bar{\mathbf{u}}^T \left[\frac{\partial \mathbf{S}}{\partial \boldsymbol{\alpha}}\right] + \bar{\mathbf{w}}^T \left[\frac{\partial \mathbf{F}}{\partial \boldsymbol{\alpha}}\right] + \bar{\mathbf{z}}^T \left[\frac{\partial \mathbf{M}}{\partial \boldsymbol{\alpha}}\right]$$

Coupled adjoint equation

$$\bar{\mathbf{u}} = \left[\frac{\partial J}{\partial \mathbf{u}}\right]^T + \left[\frac{\partial \mathbf{S}}{\partial \mathbf{u}}\right]^T \bar{\mathbf{u}} + \left[\frac{\partial \mathbf{M}}{\partial \mathbf{u}}\right]^T \bar{\mathbf{z}}$$

$$\bar{\mathbf{w}} = \left[\frac{\partial J}{\partial \mathbf{w}}\right]^T + \left[\frac{\partial \mathbf{S}}{\partial \mathbf{w}}\right]^T \bar{\mathbf{u}} + \left[\frac{\partial \mathbf{F}}{\partial \mathbf{w}}\right]^T \bar{\mathbf{w}}$$

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- From converged primal FSI

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Each term obtained through reverse AD

$$\bar{\mathbf{w}} = \left[\frac{\partial J}{\partial \mathbf{w}}\right]^T + \left[\frac{\partial \mathbf{S}}{\partial \mathbf{w}}\right]^T \bar{\mathbf{u}} + \left[\frac{\partial \mathbf{F}}{\partial \mathbf{w}}\right]^T \bar{\mathbf{w}}$$

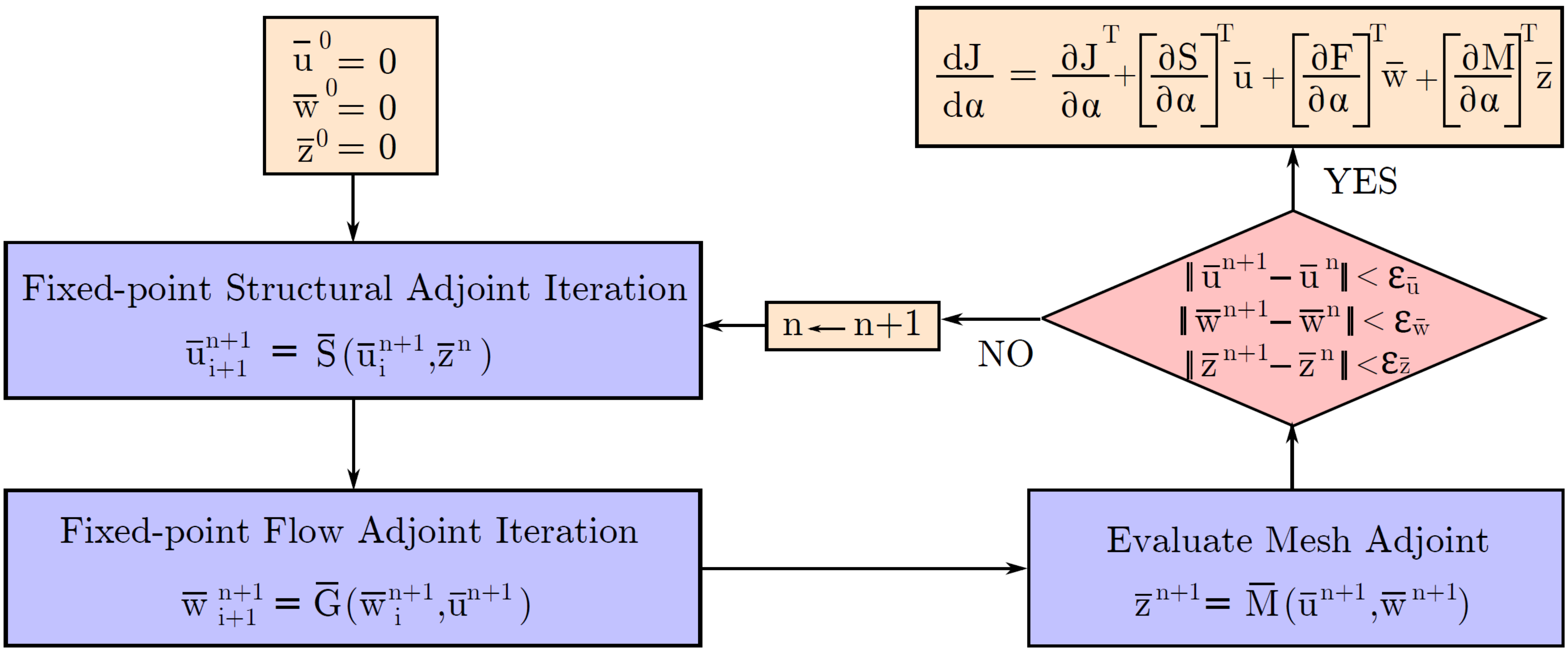
F & S equations iterated with source terms

$$\bar{\mathbf{z}} = \left[\frac{\partial J}{\partial \mathbf{z}}\right]^T + \left[\frac{\partial \mathbf{S}}{\partial \mathbf{z}}\right]^T \bar{\mathbf{u}} + \left[\frac{\partial \mathbf{F}}{\partial \mathbf{z}}\right]^T \bar{\mathbf{w}}$$

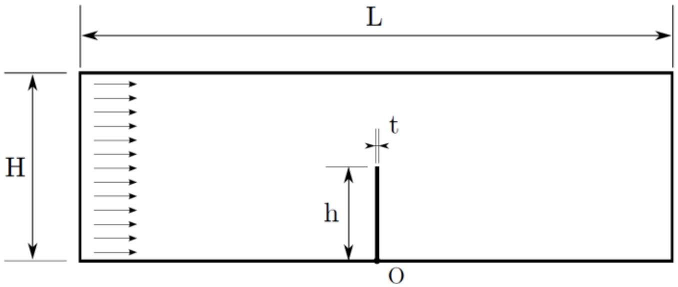
Mesh equation, direct solution

- From converged primal FSI

Block Gauss Seidel solution



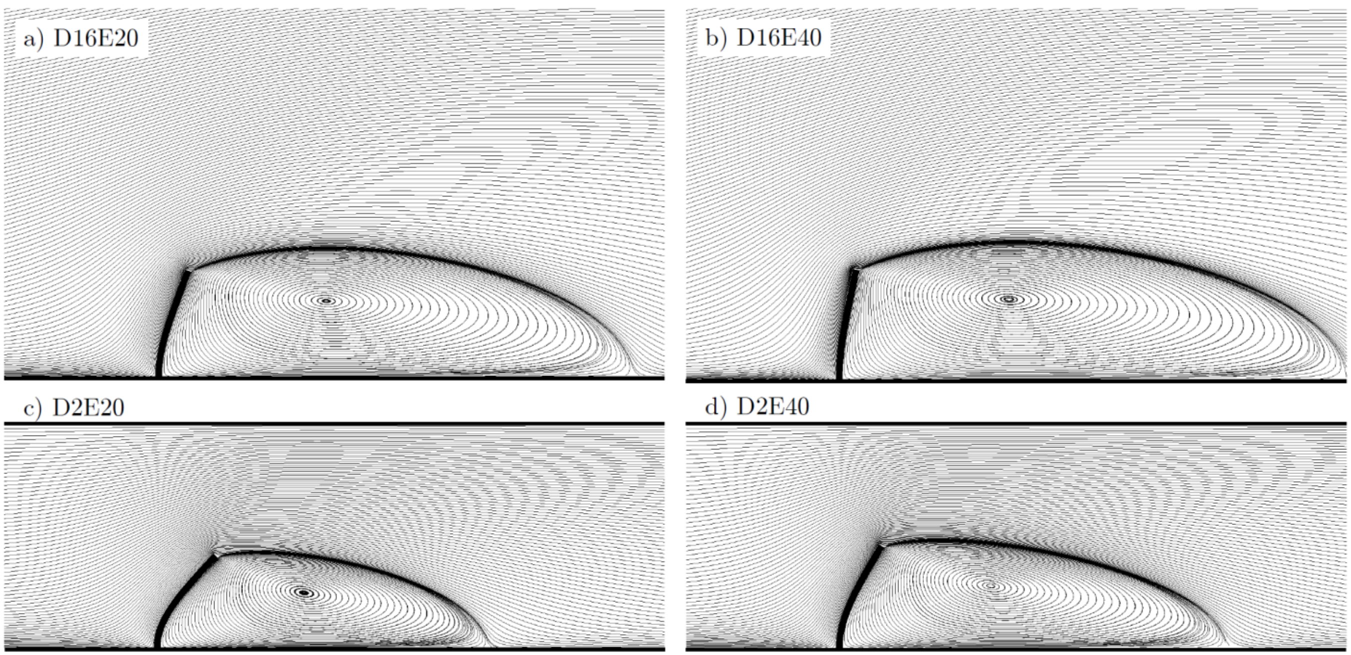
Example: A very flexible wall in a choked flow



Re_h=10

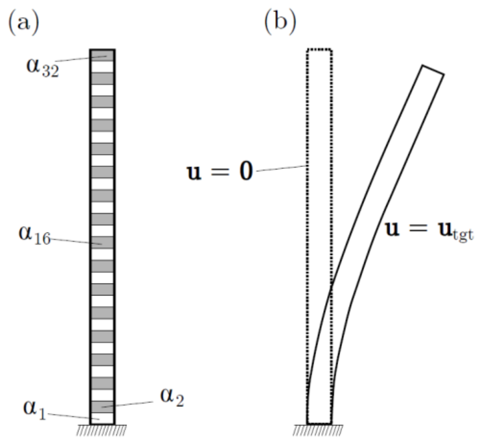
Test case	E_{ref}	ν
Very flexible wall (E20)	20 kPa	0.4
Flexible wall (E40)	40 kPa	

Test case	H	L
Wide channel (D16)	16·h	48·h
Narrow channel (D2)	2·h	

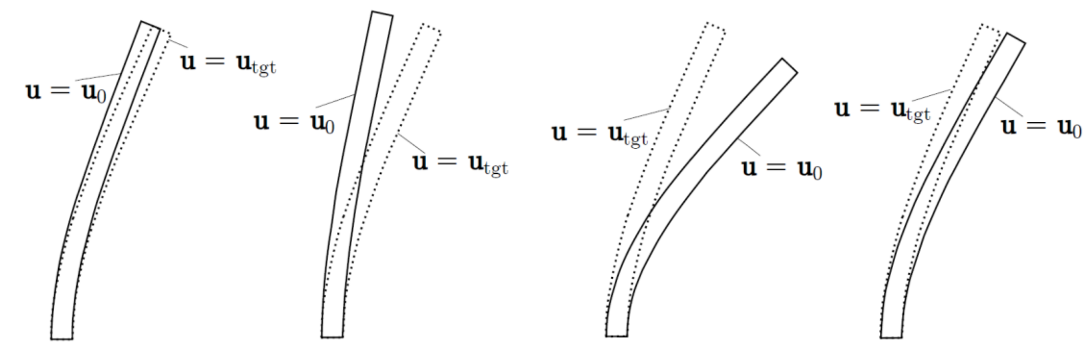


Inverse problem

- Goal a target shape in FSI equilibrium
- 32 design variables



Four reference equilibria against target shape



(a) D16E20 (b) D16E40 (c) D2E20 (d) D2E40

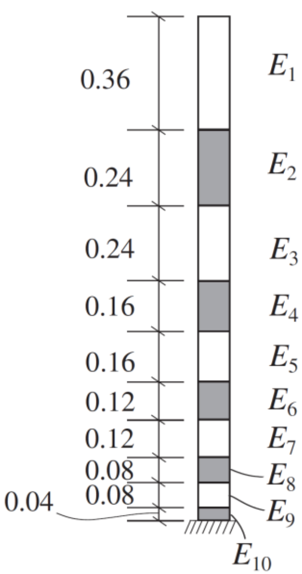
- Two costs:

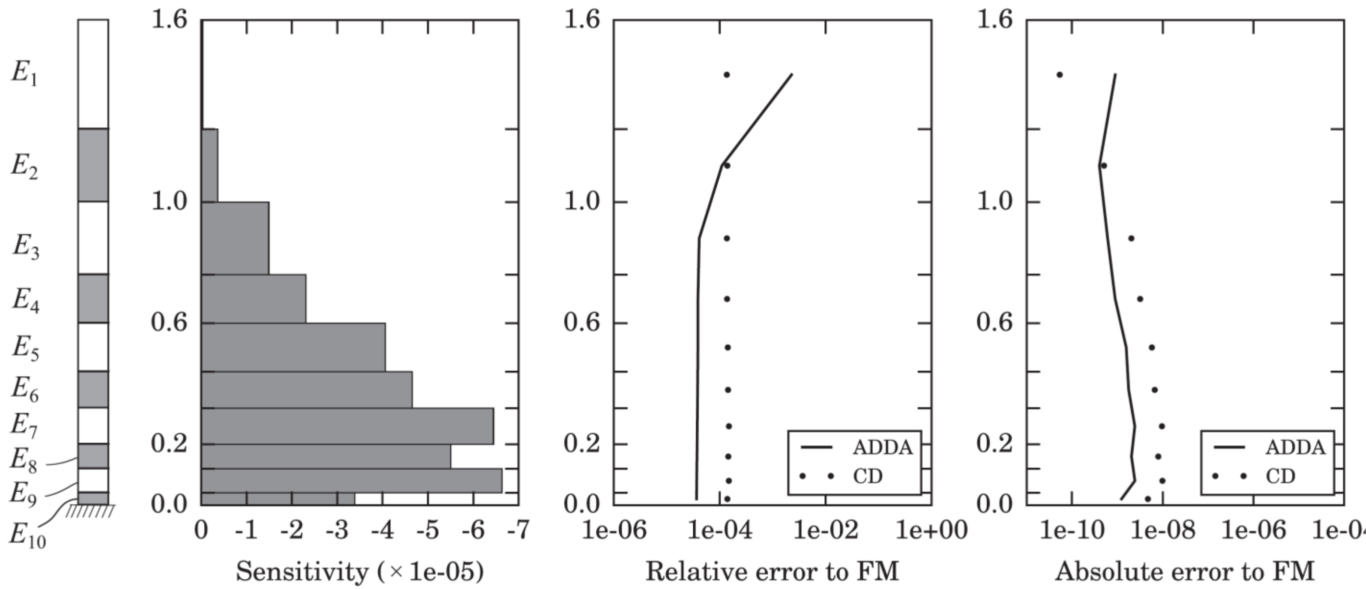
$$J_{uncons}(\mathbf{u}) = (\mathbf{u} - \mathbf{u}_{tgt})^T \mathbf{C} (\mathbf{u} - \mathbf{u}_{tgt})$$

$$J_{cons}(\mathbf{u}, \boldsymbol{\alpha}) = (\mathbf{u} - \mathbf{u}_{tgt})^T \mathbf{C} (\mathbf{u} - \mathbf{u}_{tgt}) + k \left(\frac{\sum_{i=1}^{n_\alpha} \alpha_i}{n_\alpha} - 1.0 \right)^2$$

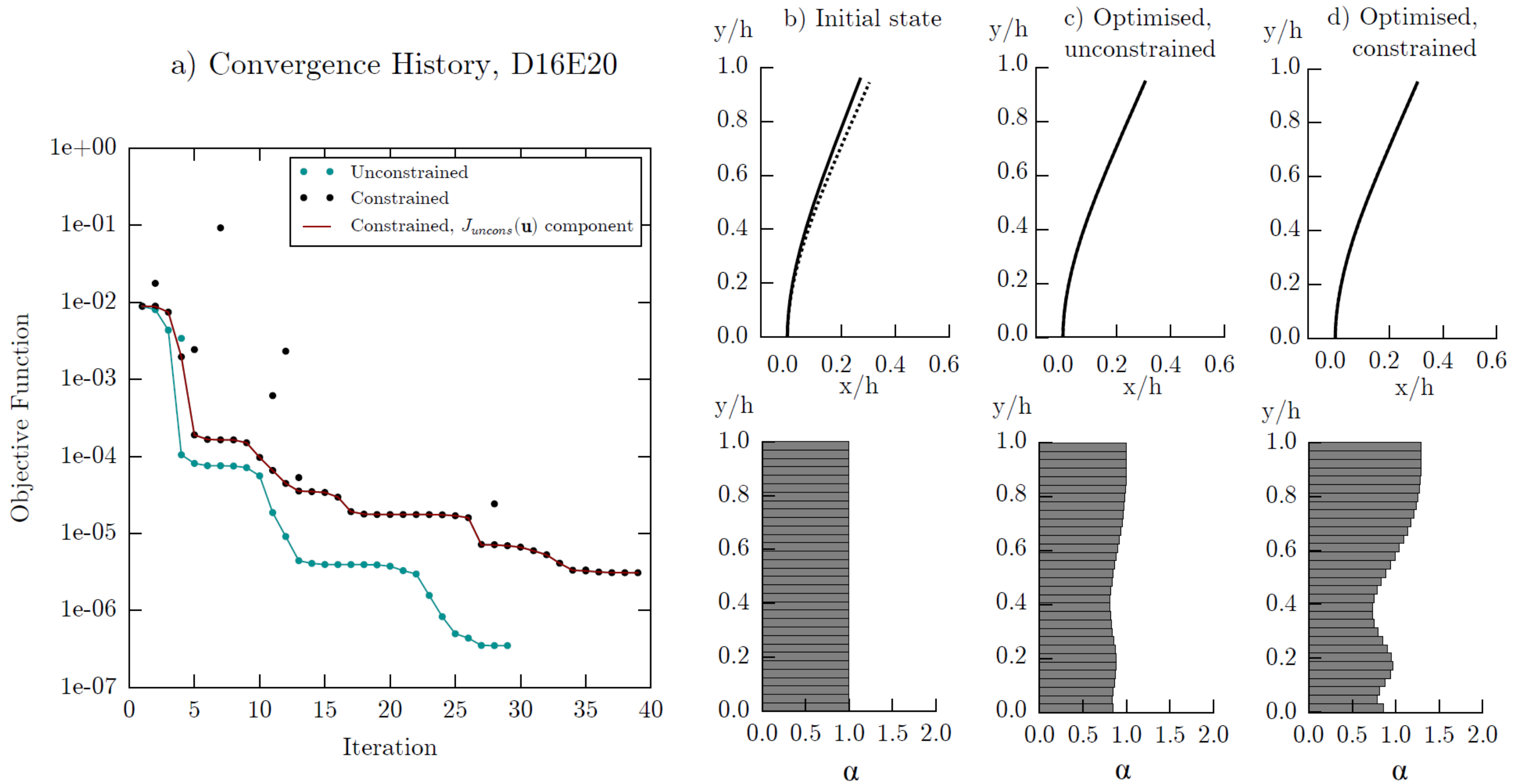
Verification of coupled sensitivities

- Comparison with forward mode and central differences
- Only for 10 zones



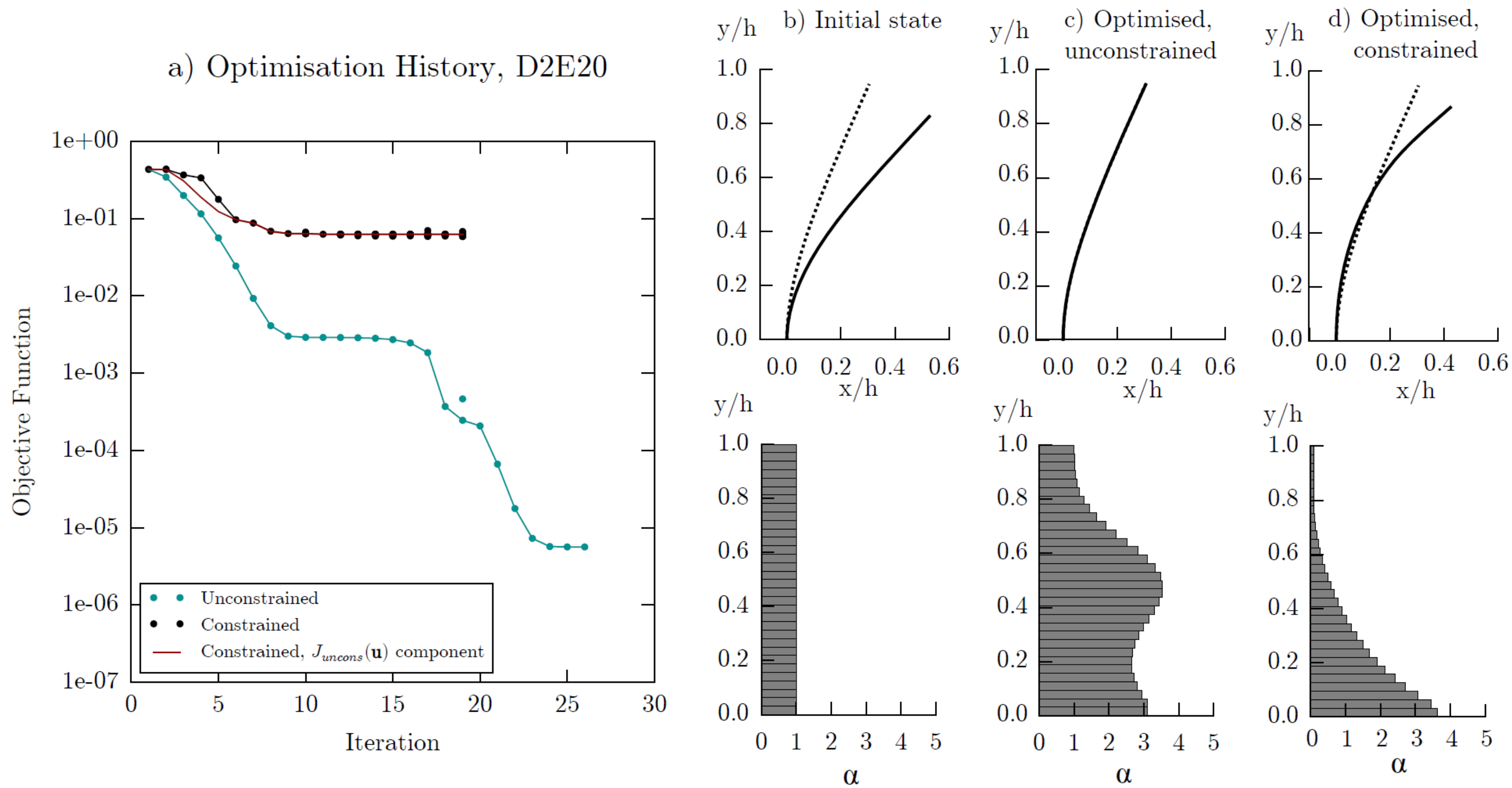


Optimal stiffness distribution



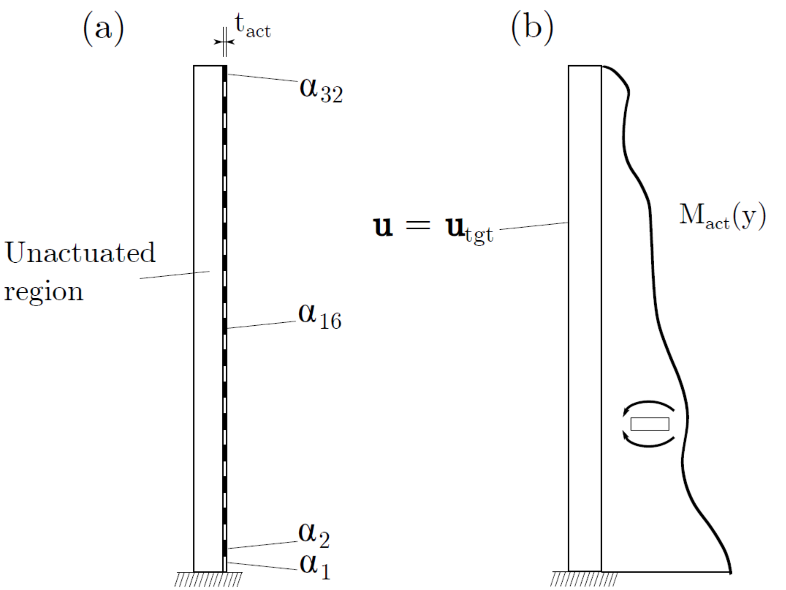
Objective Function	Obj. Function Evaluations	Gradient Evaluations	$\min J(\mathbf{u})$	$\Delta\boldsymbol{\alpha}_{ref}$
$J_{uncons}(\mathbf{u})$	30	29	3.50E-07	-10.33%
$J_{cons}(\mathbf{u},\boldsymbol{\alpha})$	48	39	3.09E-06	+0.00%

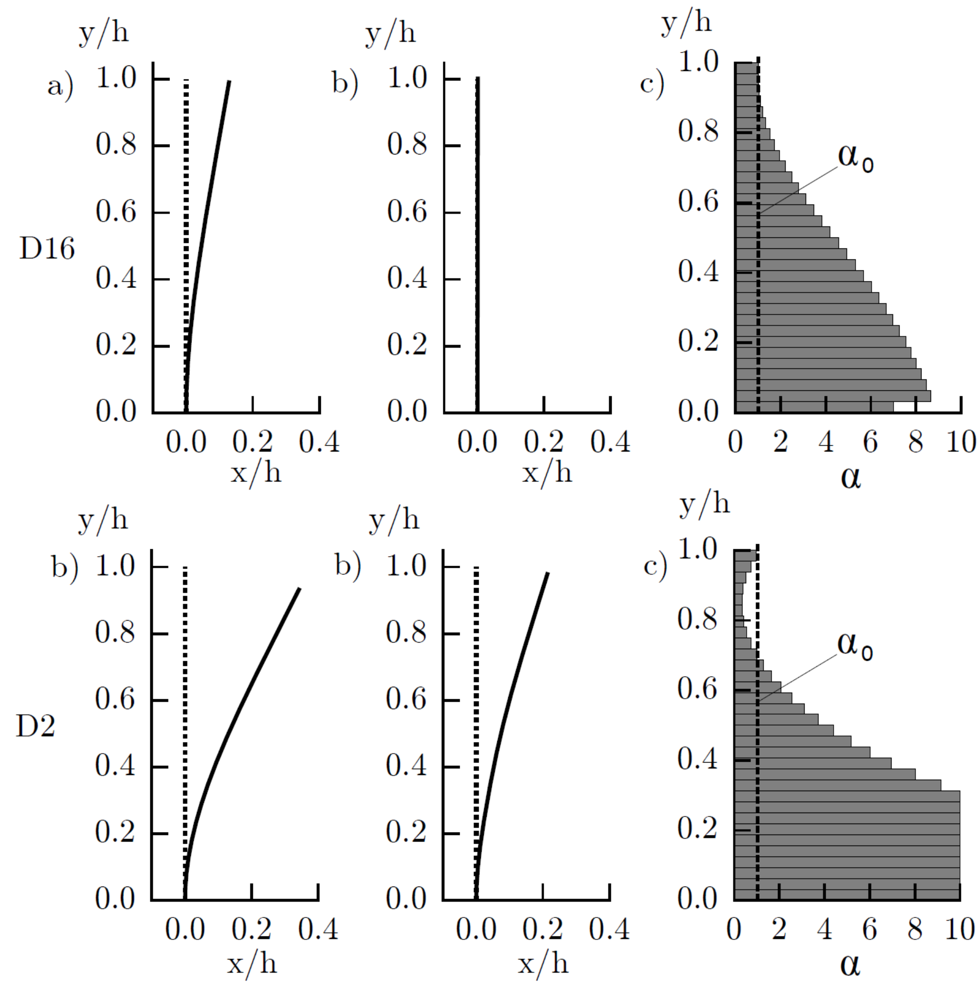
Optimal stiffness distribution



Objective Function	Obj. Function Evaluations	Gradient Evaluations	$\min J(\mathbf{u})$	$\Delta\boldsymbol{\alpha}_{ref}$
$J_{uncons}(\mathbf{u})$	27	26	5.64E-06	+144.29%
$J_{cons}(\mathbf{u},\boldsymbol{\alpha})$	111	19	6.27E-02	+0.01%

Optimal electromechanical actuation

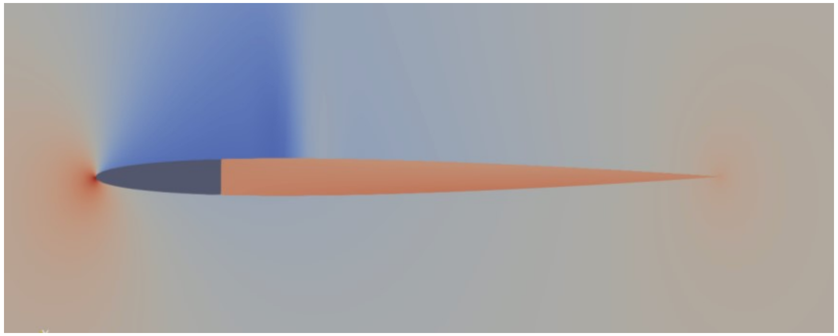


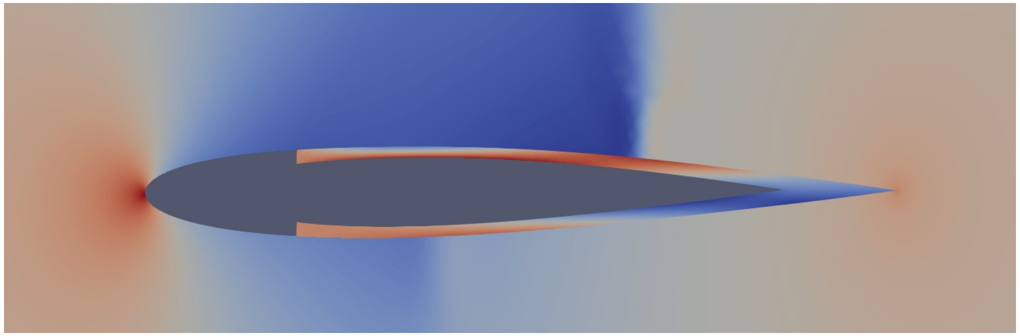


Test Case	$J(\mathbf{u})$ Evals	$\nabla J(\mathbf{u})$ Evals	$J(\boldsymbol{\alpha}_0)$	$J(\boldsymbol{\alpha}_{opt})$
D2	39	39	8.06E-01	2.96E-01
D16	23	22	1.08E-01	1.19E-04

On-going

- Aero shape optimization with structure in the loop:



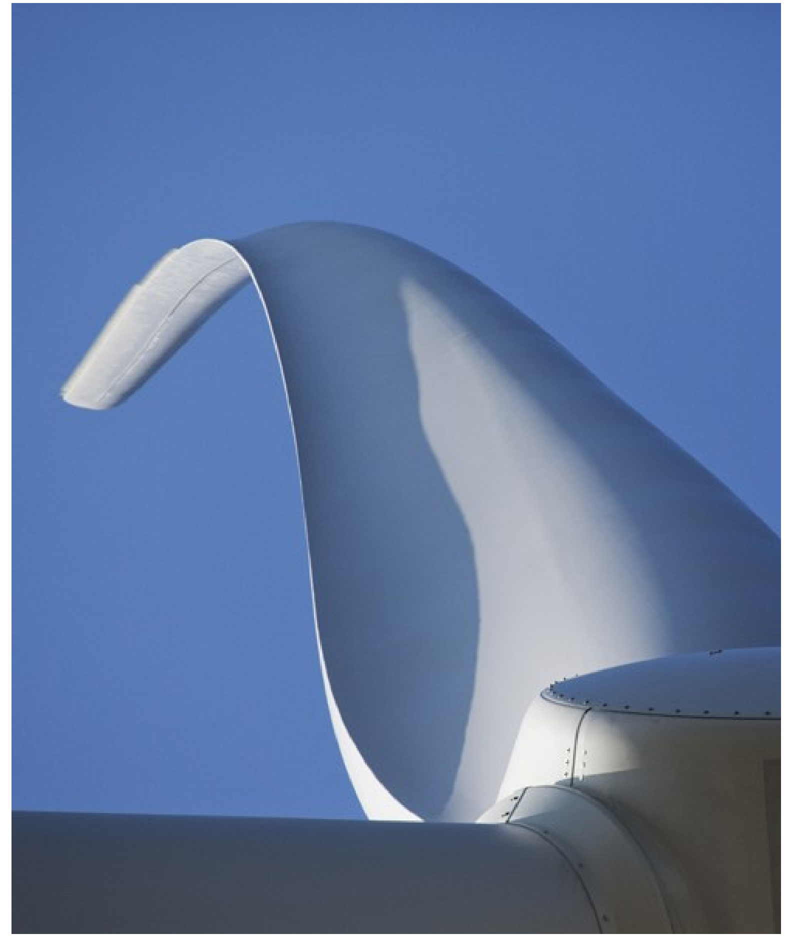


- Up-scaling the problems (e.g. towards the design of 3D-printed aeroelastic wings)
- Library of RBF interpolation routines
- Python-wrapper with beam skeleton:

From RWTH Aachen

Current To-Do list

- Solid-mechanics-friendly linear algebra solver
- Higher-order FE
- Robust mesh deformation
- Directional properties (composite materials)
- Expanded library of elements and BCs
- Time-domain FSI adjoints
- General multiphysics architecture



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