Anisotropic Mesh Adaptation with the (Py)AMG Library

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3rd Annual SU2 Developers Meeting
Research Interests of the Gamma3 Team

Surface meshing (triangular, hexahedral) (software: BLSURF, Hexotic)

Volume meshing (Software: GHS3D, Hexotic)

Boundary layer meshing (AMG_BL, Bloom)

Adaptivity: Error estimates, Anisotropic mesh generation
INRIA/Stanford collaboration

• Informal collaboration for a long time (10 years)

• 2014 : Collaboration of 1\textsuperscript{st} AIAA Sonic Boom Workshop
  
  • First anisotropic computations with SU2/AMG, joint work with ONERA
  • Inviscid runs

• 2016 : Internal use during V. Menier’s post doc

First discussions of exposing an interface for SU2
Outline

• Background on anisotropic mesh adaptation
  • Metric-based mesh generation
  • Error estimates
  • Unique cavity-based operators
  • Numerical examples : RANS Adaptation

• PyAMG interface and SU2
  • Python bindings
  • Numerical adaptive examples
Motivations

• Physical phenomena have strong anisotropic components
• Uniform meshes are not optimal in term of sizes and directions

• Anisotropic mesh adaptation is a way to optimize the ratio CPU time versus accuracy
Example: Direct sonic boom

Classical CFD

Adaptive CFD

NEAR-FIELD REGION

MID-FIELD REGION

PROPAGATION

FAR-FIELD REGION

GROUND

10 km

15 km

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Example: Direct sonic boom

- Initial mesh 415,535 vertices
- Volume ratio $[5 \times 10^{11}, 4.7 \times 10^{10}]$
- Size ratio $10^9$
- Final adapted 3,299,676 vertices
- **CPU time**: 4 hours
- If done with uniform refinement: $10^{18}$ tetrahedra
- If done with uniform mesh: **1000 years of computation**

**Mesh adaptation**:
- Early capturing of physical phenomena
- Second order convergence on flows with shocks
- Capture all the scales of the flow
Mesh Adaptation is a non-linear problem

\[(H_0, S_0^0)\]

\[(H_i, S_i^0)\]

\[(H_i, S_i, H_i)\]

\[(H_{i+1}, S_{i+1}, H_{i+1})\]

\[(H_0, S_0^0)\]

\[S_i\]

\[S_{i+1}\]

\[H_{i+1}\]

\[H_i\]

\[M_i\]

\[(Py)AMG\]

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Metric-based framework

• **Main idea** : change the distance and volume computation used the mesh generator [George, Hecht and Vallet., Adv. Eng. Software 1991]

• **Fundamental concept** : The notion of metric Riemannian metric space
  • Euclidean Metric space
  \[ \ell_M(ab) = d_M(a, b) = \|ab\|_M \]
  \[ \langle u, v \rangle_M = \langle u, Mv \rangle = \langle u, Mv \rangle \]
  • Riemannian Metric space
  \[ M = (M(x))_{x \in \Omega} \]
  \[ \ell_M(ab) = \int_0^1 \sqrt{t M(a + t ab)} \, ab \, dt \]
Metric-based error estimates

• **Scope**: From numerical solution derive a metric-field to drive adaptivity

  [Venditti and Darmofal, JCP 2003], [Jones et al., AIAA 2006], [Power et al., CMA 2006], [Wintzer et al., AIAA 2008], [Leicht and Hartmann, JCP 2010],

• **Multi-scale error estimates**:
  • Derive the ‘best’ mesh to compute the characteristics of a given solution $W$
  • Optimal control of the interpolation error in $L^p$ norm:

\[
\|W - \Pi_h W\|_{L^p(\Omega_h)}
\]

• **Goal-oriented error estimates**
  • Derive the ‘best’ mesh to observe a given functional $j(w) = (g, w)$
  • Optimal control of the interpolation error in $L^p$ norm:

\[
|j(W) - j(W_h)| \approx \|\nabla W^* \cdot (F(W) - \Pi_h F(W))\|_{L^2(\Omega_h)}
\]

Observation

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Anisotropic mesh generation

- Generate a **unit mesh** w.r.t. the input metric field.
- Local mesh operations are performed:
- Standard mesh operators: point insertion/deletion, edge swap/collapse, etc.
- In AMG: all these operations are embedded in **one cavity-based operator**

\[
\ell_M(\vec{v}) = 1
\]
Standard meshing operators

- Move point
- Swap edge
- Collapse edge
- Point insertion

• All these operators are applied sequentially and are monitored by a quality function

• Step 1: Generate a unit mesh
  • Split long edges in the metric
  • Collapse small edges in the metric

• Step 2: Optimization
  • Perform point smoothing and swaps
Unique cavity-based operator

[Loseille and Löhner, AIAA 2010, Loseille et Menier, IMR 2013]

- **In PyAMG:**
  - Each mesh operation corresponds to a node (re)insertion

- **Unified**: all-in-one
- **Generalized**: multiple-operators at once
  - Do either line, surface or volume
  - Handle non-manifold geometries
  - Easy to maintain and update
  - Uniform speed for all operators
HL-CRM

• Mach 0.2, 16 degrees, 3,2 Million
• Meshing process: start from a coarse mesh
• Geometry: P3 curved mesh (built from CAD)

Software packages

• Wolf flow solver: FV/FE, HLLC scheme, Implicit, SA-Neg
• Goal oriented RANS error estimates: lift functional
• Fully anisotropic: no quasi-structured boundary layer mesh inserted

• Final mesh: 13 M vertices
• 10 Adaptations
Workshop grids:
- Coarse: 8 Million vertices
- Medium: 26 Million vertices
- Fine: 70 Million vertices
- X-Fine: 206 Million vertices

Adaptive grids: 10 adaptations
- Level 1: 3.1 Million vertices
- Level 2: 13 Million vertices
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Challenges of exposing adaptivity

• Automaticity: Minimal user intervention/parameters
• Geometry: surface approximation/projection/CAD
• Mesh Quality/Anisotropy ↔ Solver Stability/Accuracy
• Error estimates: theoretical developments are still on-going

Typical issues with CFD solvers

• Associativity of boundary conditions to initial Geometry/CAD are lost ..
• Restart capabilities
• Strong solvers: stability and convergence on highly anisotropic elements
PyAMG: technical details

• Use of Python: Mesh, Solution and Options are dictionaries

• Expose first a reduced set of capabilities
  • P3 High-order geometry prior to expose CAD projection
  • Multi-scale error estimate prior to expose Goal-oriented
  • Fully anisotropic prior to expose adaptive quasi-structured BL

• Have a reduced number of user-parameters for mesh adaptation
  • Accuracy level
  • Gradation of the mesh
  • Sensor

• Full integration with SU2 in config file
PyAMG website

• [https://pyamg.saclay.inria.fr](https://pyamg.saclay.inria.fr)

pyAMG
A fast, robust mesh adaptation software for 2D and 3D complex geometries, all wrapped in Python. pyAMG is developed at Inria and released for free for non-commercial use.

pyAMG Examples
This page presents some examples of mesh adaptation using pyAMG.

Stand-alone examples
- Simple 2D uniform refinement
- Control of the interpolation error of an analytical function
- Mesh adaptation according to a velocity field

Examples using SU2
These examples require to have the pyAMG/SU2 interface installed.

- Adaptation of a 2D NACA airfoil
- Adaptive CFD RANS wing
PyAMG : example of mesh definition

```python
import pyamg
mesh = {}
mesh['xy'] = [[0,0], [1,0], [1,1], [0,1]]
mesh['Triangles'] = [[1,2,3,0], [3,4,1,0]]
print "Writing initial_mesh.mesh"
pyamg.write_mesh(mesh, "initial_mesh.mesh")
remesh_options = {'hmax':0.5}
print "Refining mesh and writing refined_mesh.mesh"
adapted_mesh = pyamg.adapt_mesh(mesh, remesh_options)
pyamg.write_mesh(adapted_mesh, "refined_mesh.mesh")
```

- Solutions is stored in key ‘sensor’
- Option-related keywords include: ‘hmax’, ‘hmin’, ‘Lp’, ‘gradation’
PyAMG : Example of mesh adaptation

```python
# Define remeshing options
remesh_options = {}
remesh_options['Lp'] = 2
remesh_options['gradation'] = 1.5
remesh_options['target'] = 20000

# Adapt the mesh, perform 5 iterations
print "  Anisotropic adaptation : iteration 1"
msh3d_aniso = pyamg.adapt_mesh(msh3d, remesh_options)

for ite in range(2,5):
    print "  Anisotropic adaptation : iteration %d " %(ite)
msh3d_aniso['sensor'] = create_sensor(msh3d_aniso)
msh3d_aniso = pyamg.adapt_mesh(msh3d_aniso, remesh_options)

# Output final mesh
pyamg.write_mesh(msh3d_aniso,"cube_aniso.meshb")
```
PyAMG-SU2: Example

```
% ---------------- MESH ADAPTATION PARAMETER ----------------%

% Mesh size parameters
ADAP_SIZES= (2000, 4000, 8000)

% Number of iterative loops performed for each prescribed size
ADAP_SUBITE= (2, 2, 2)

% Number of CFD iterations for each mesh size
ADAP_EXT_ITER= (1000, 1000, 1000)

% Prescribed residual reduction for each mesh size
ADAP_RESIDUAL_REDUCTION= (3, 3, 3)

% Sensor used for mesh adaptation (MACH, PRES, or MACH_PRES)
ADAP_SENSOR= MACH

% Max and min edge sizes
ADAP_HMAX= 200
ADAP_HMIN= 1e-8

% Prescribed mesh gradation
ADAP_HGRAD= 1.3

% Output adapted mesh
MESH_OUT_FILENAME= M6_adap.su2

% Final adapted restart solution
RESTART_FLOW_FILENAME= M6_adap.dat
```

- Everything is contained in the config file
- PyAMG bindings are transparent to the user

```
$ python mesh_adaptation_amg.py -f adap_ONERAM6.cfg
```
PyAMG-SU2 : Behind the scene

- Convert SU2 data structures into Python dictionaries
- Expand/Split MARKERS to be as close as possible of the most detailed geometry description (CAD Patch)

2 BC : body + inlet

CFD Boundary condition

83 CAD Patches

Good practice
PyAMG-SU2 : current status

• Inviscid and RANS with frozen boundary layers are support
• Support to fully unstructured boundary layers is on-going
• The adaptive script is still a custom branch of SU2
PyAMG-SU2 : Inviscid CRM

- Mach 0.85, 2.133 degrees, Inviscid
- Meshing process: start from a coarse mesh
- Geometry: P3 curved mesh (built from CAD)

Software packages

- SU2: JST, MG, FMGRES
- Multi-scale error estimate: Mach Number
- Final mesh 3,019,832 vertices
PyAMG-SU2: Inviscid CRM
PyAMG-SU2 : RANS Case

• Mach 2.0, 10 degrees, 1.65 Million
• Meshing process : start from a coarse mesh
• Geometry : P3 curved mesh (built from CAD)

Software packages

• SU2 : SA, JST, FMGRES
• Multi-scale error estimate : Mach Number
• Frozen boundary layer
• Final mesh 1 343 684 vertices (outside BL)
PyAMG/SU2: RANS case
Conclusion

• Beta version of adaptive mesh generation in 2D/3D
• Error estimates, interpolation, surface, volume mesh generations
• Inviscid runs, RANS adaptations

On-going work

• Increase the set of documented adaptive test cases in 2D/3D
• Detail best practice to start a new adaptive computation (Geometry)

Stay updated

• Visit pyamg.saclay.inria.fr for more details
• Mailing list : send an email to pyamg-request@inria.fr with subscribe as topic
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• Unstructured Grid Adaptation Working Group
  https://github.com/UGAWG

• Visualization software: ViZiR team (INRIA)
  https://vizir.inria.fr/
Thank you for your attention!

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