Anisotropic Mesh Adaptation with the (Py)AMG Library

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And
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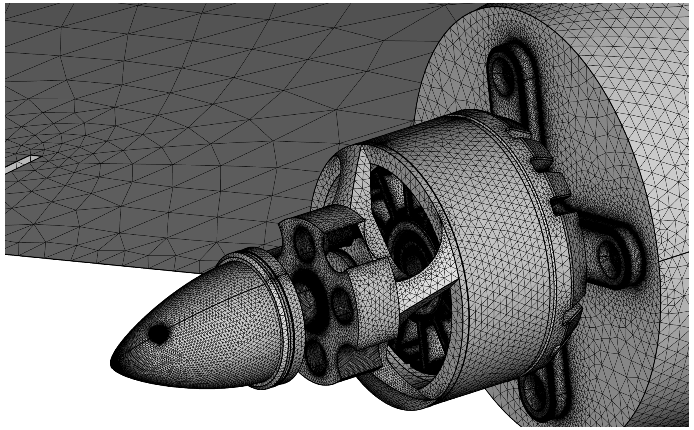
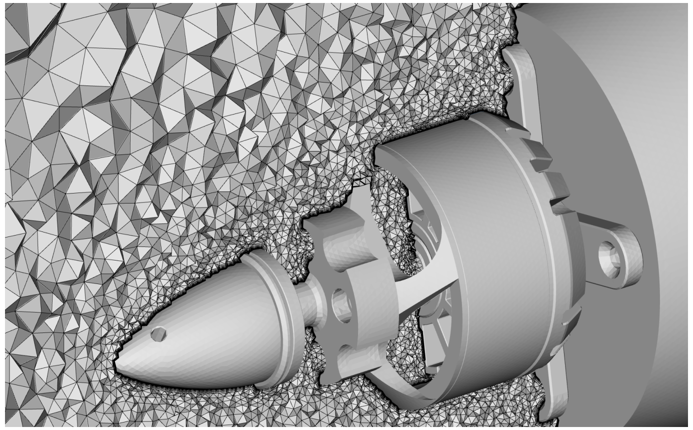
3rd Annual SU2 Developers Meeting

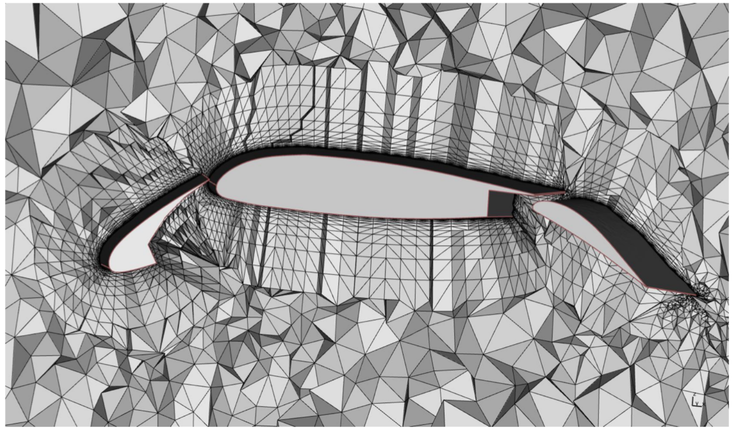
Research Interests of the Gamma3 Team



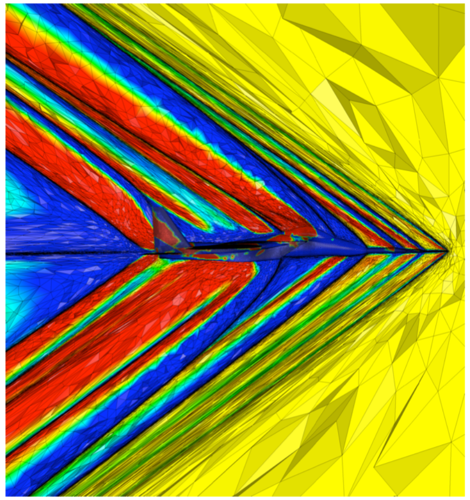
Surface meshing
(triangular, hexahedral)
(software: BLSURF, Hexotic)

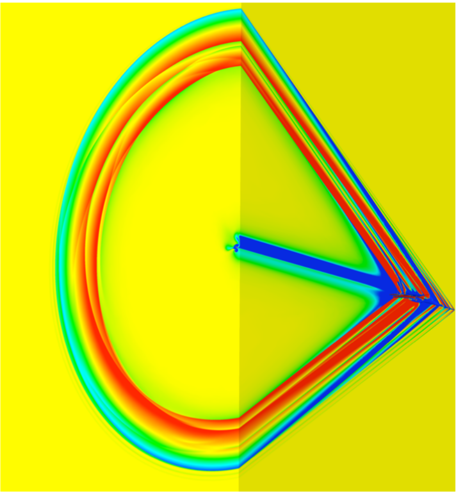


Volume meshing
(Software: GHS3D, Hexotic)



Boundary layer meshing
(AMG_BL, Bloom)





Adaptivity : Error estimates, Anisotropic mesh generation

INRIA/Stanford collaboration

- Informal collaboration for a long time (10 years)
- 2014 : Collaboration of 1st AIAA Sonic Boom Workshop
  - First anisotropic computations with SU2/AMG, joint work with ONERA
  - Inviscid runs
- 2016 : Internal use during V. Menier's post doc

→ First discussions of exposing an interface for SU2

Outline

- Background on anisotropic mesh adaptation
  - Metric-based mesh generation
  - Error estimates
  - Unique cavity-based operators
  - Numerical examples : RANS Adaptation

- PyAMG interface and SU2
  - Python bindings
  - Numerical adaptive examples

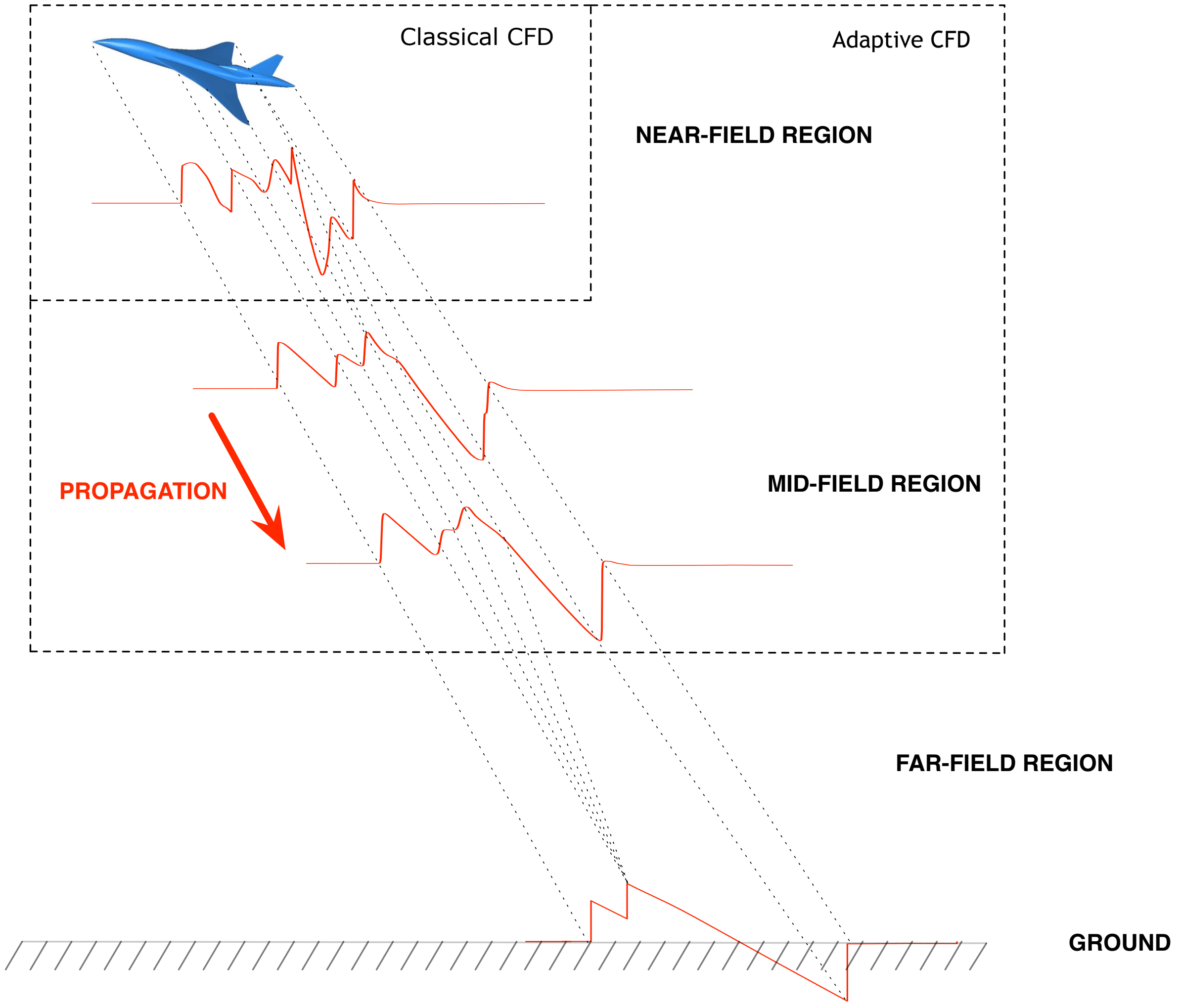
Motivations

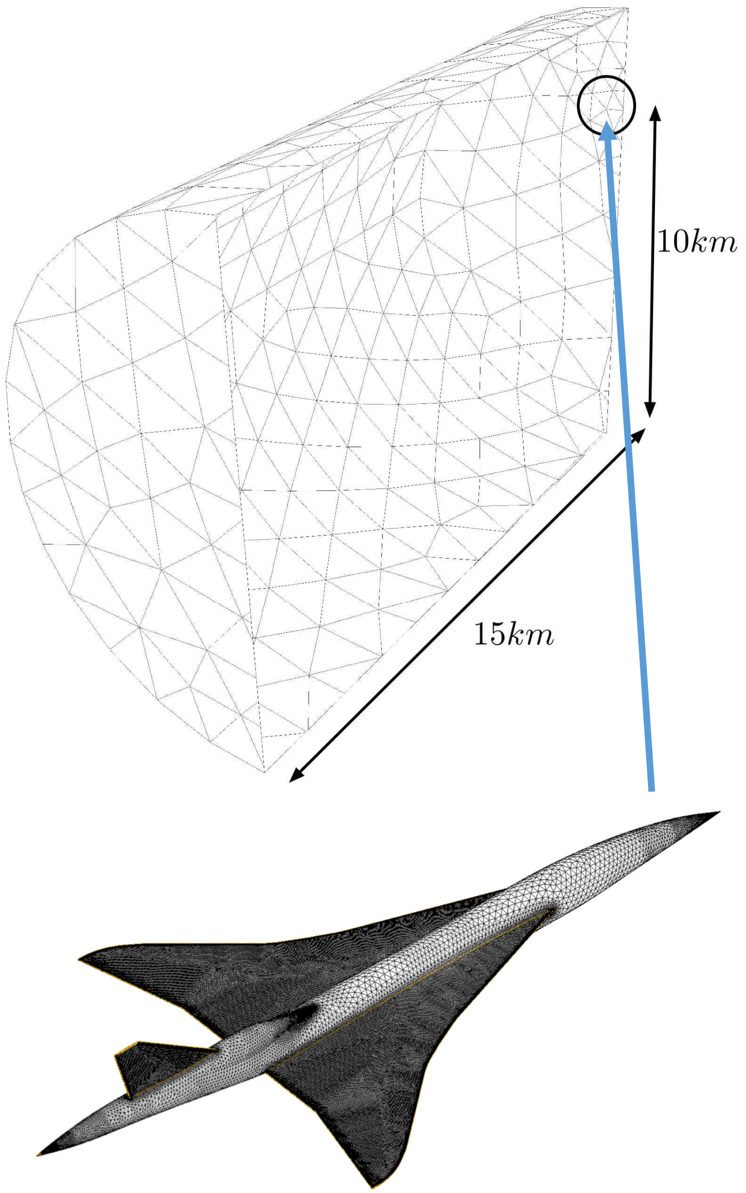
- Physical phenomena have strong anisotropic components
- Uniform meshes are not optimal in term of sizes and directions

- Anisotropic mesh adaptation is a way to optimize the ratio

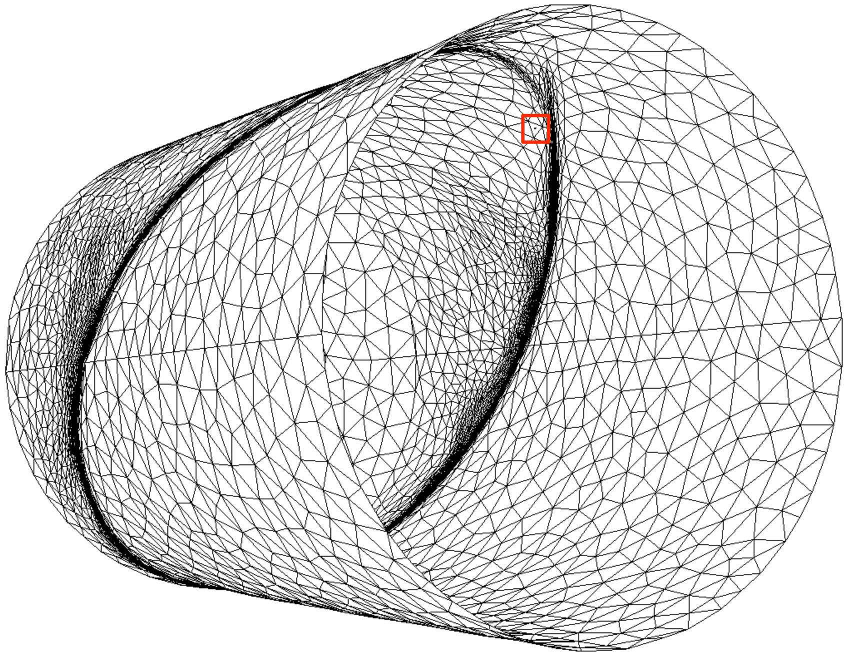
CPU time versus accuracy

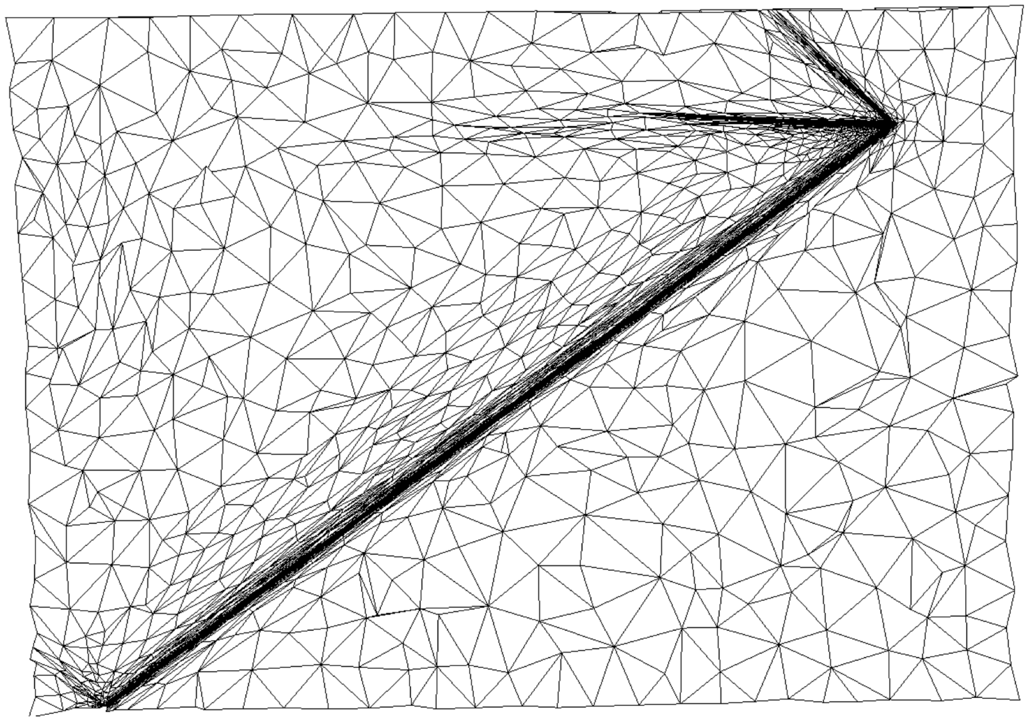
Example : Direct sonic boom



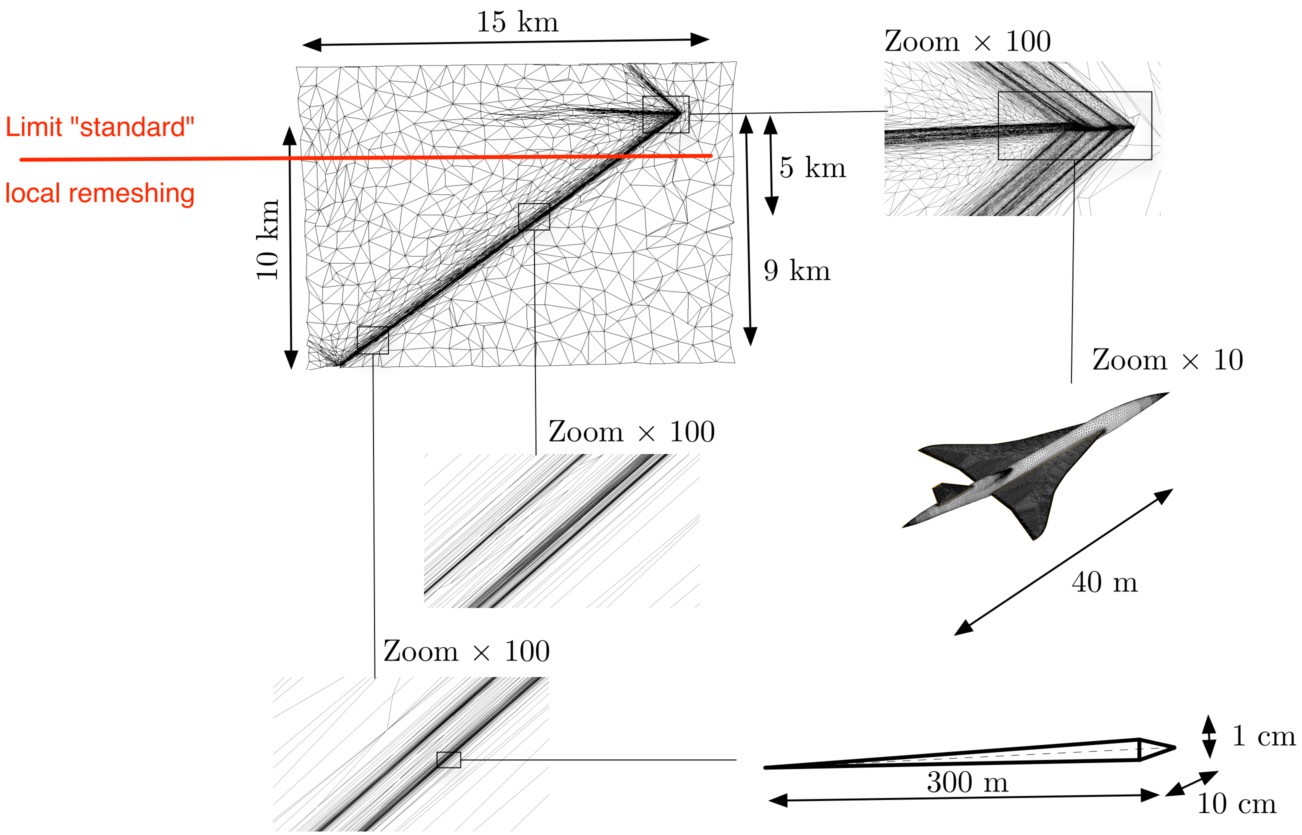


Example : Direct sonic boom





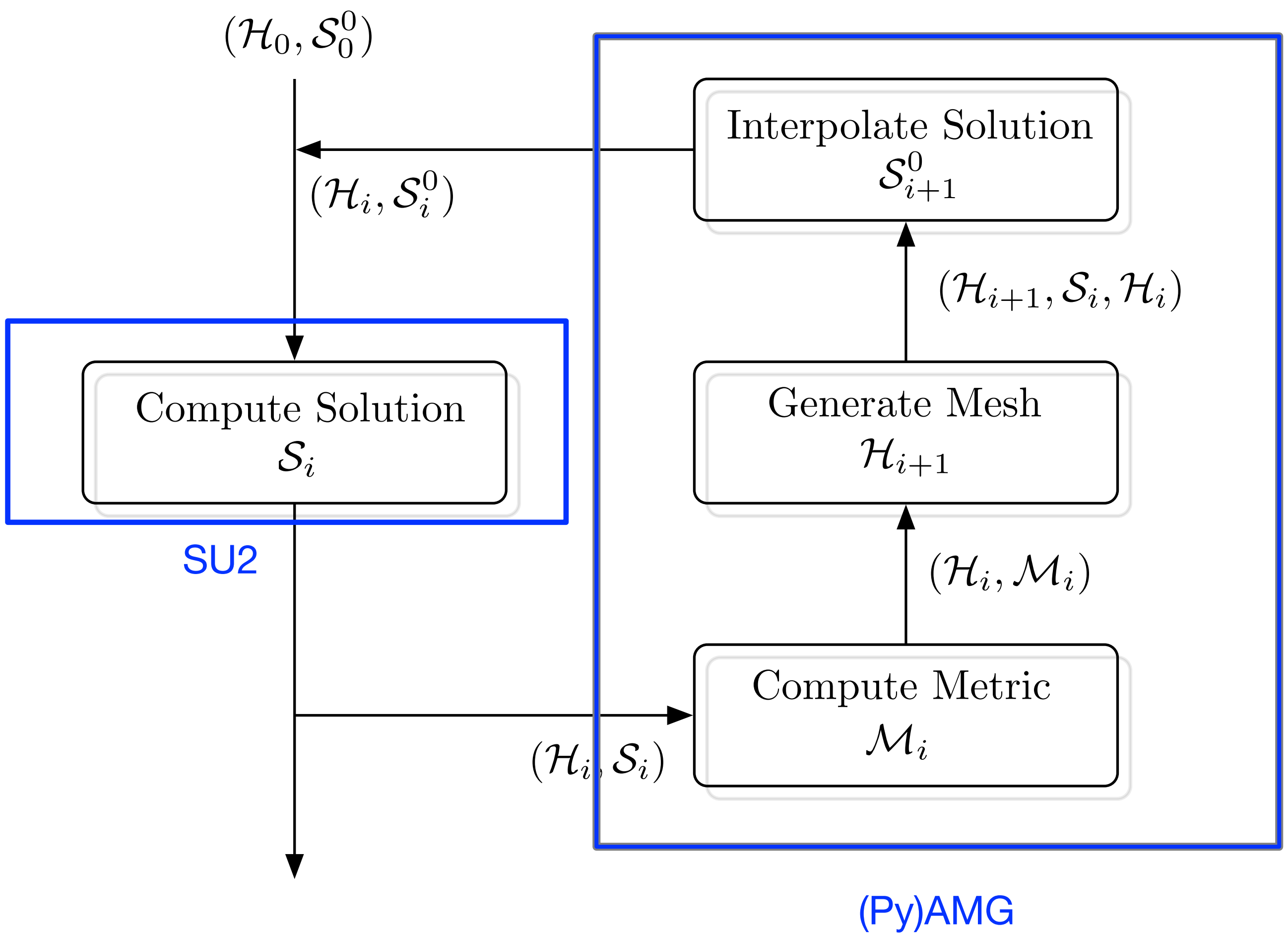
- Initial mesh 415 535 vertices
- Volume ratio [5.e^{-11} 4.7e^{10}]
- Size ratio 10^9
- Final adapted 3 299 676 vertices

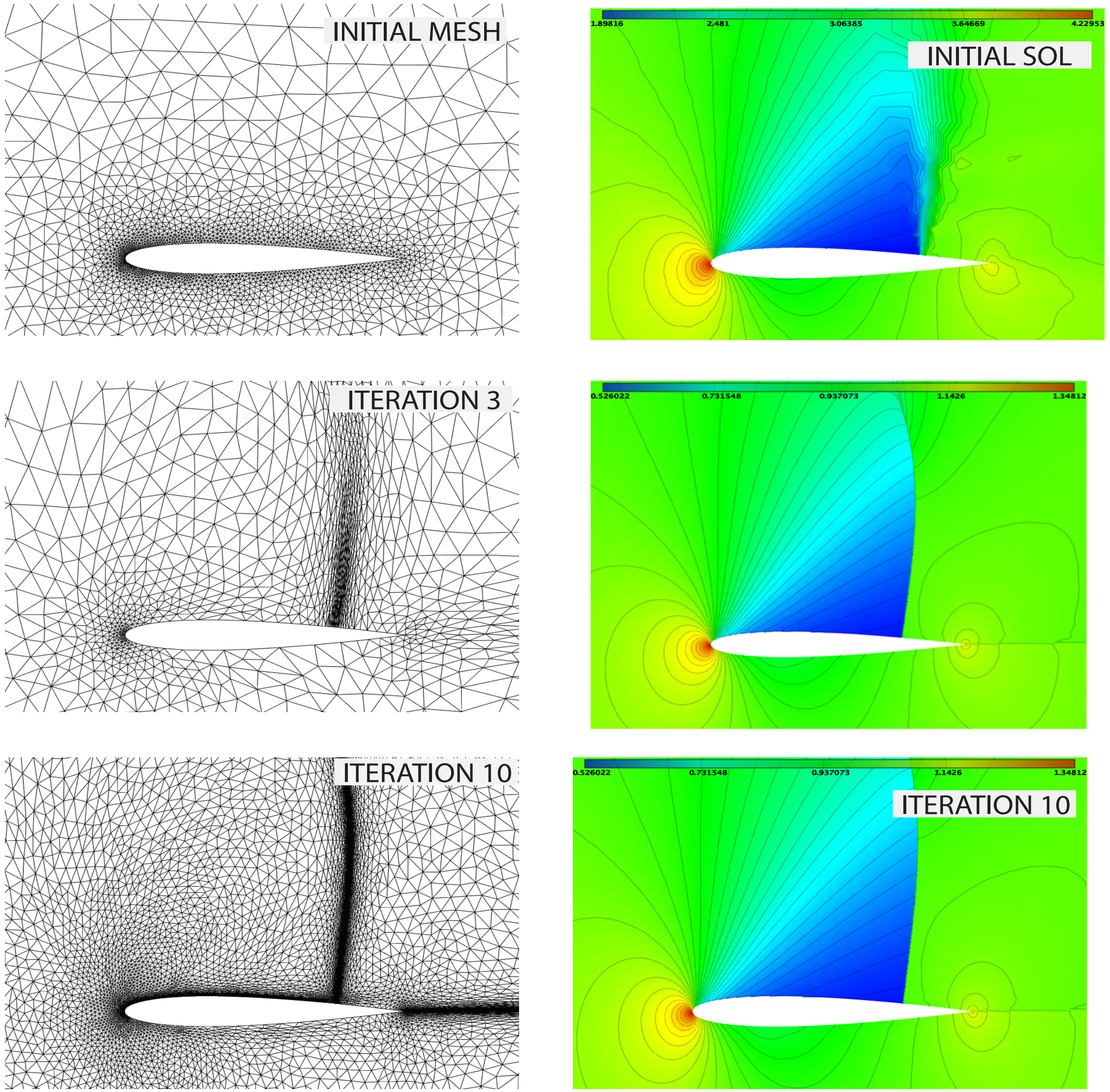


- CPU time : 4 hours
- If done with uniform refinement : 10^{18} tetrahedra
- If done with uniform mesh : 1000 years of computation

- **Mesh adaptation :**
  - Early capturing of physical phenomena
  - Second order convergence on flows with shocks
  - Capture all the scales of the flow

Mesh Adaptation is a non-linear problem





Metric-based framework

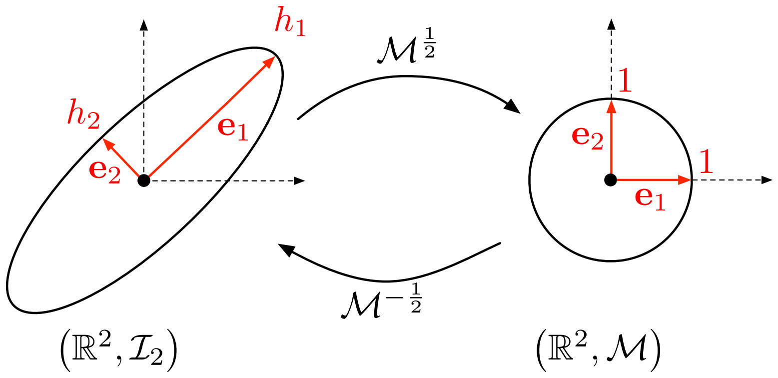
- Main idea : change the distance and volume computation used the mesh generator [George, Hecht and Vallet., Adv. Eng. Software 1991]
- Fundamental concept : The notion of metric Riemannian metric space
  - Euclidean Metric space
  - Riemannian Metric space

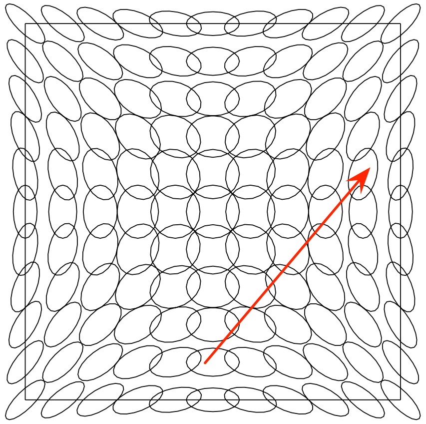
$$\ell_{\mathcal{M}}(\mathbf{ab}) = d_{\mathcal{M}}(\mathbf{a},\mathbf{b}) = \|\mathbf{ab}\|_{\mathcal{M}}$$

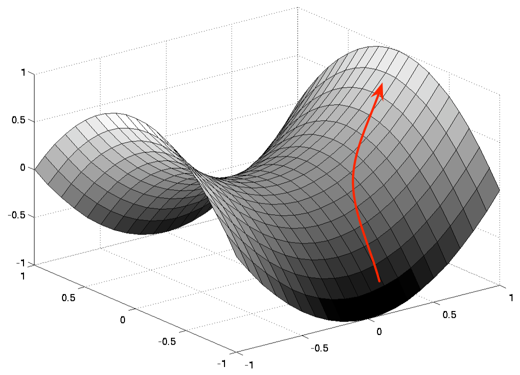
$$\langle \mathbf{u},\mathbf{v}\rangle_{\mathcal{M}} = \langle \mathbf{u}, \mathcal{M}\mathbf{v}\rangle = {}^t\mathbf{u}\mathcal{M}\mathbf{v}$$

$$\mathbf{M} = (\mathcal{M}(\mathbf{x}))_{\mathbf{x}\in\Omega}$$

$$\ell_{\mathcal{M}}(\mathbf{ab}) = \int_0^1 \sqrt{{}^t\mathbf{ab}\,\mathcal{M}(\mathbf{a}+t\,\mathbf{ab})\,\mathbf{ab}}\,\mathrm{d}t$$







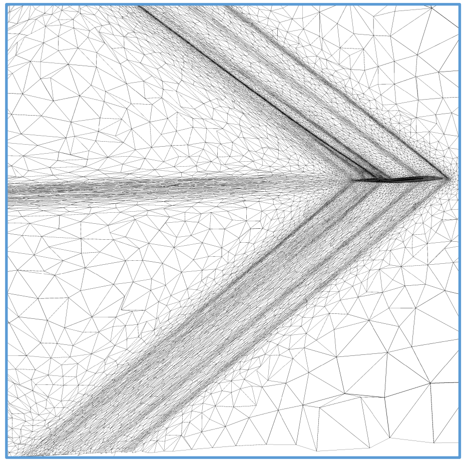
Metric-based error estimates

- Scope : From numerical solution → derive a metric-field to drive adaptivity

[Venditti and Darmofal, JCP 2003], [Jones et al., AIAA 2006], [Power et al., CMA 2006], [Wintzer et al., AIAA 2008], [Leicht and Hartmann, JCP 2010],

Multi-scale error estimates :

- Derive the ’best’ mesh to compute the characteristics of a given solution W
- Optimal control of the interpolation error in L^p norm:



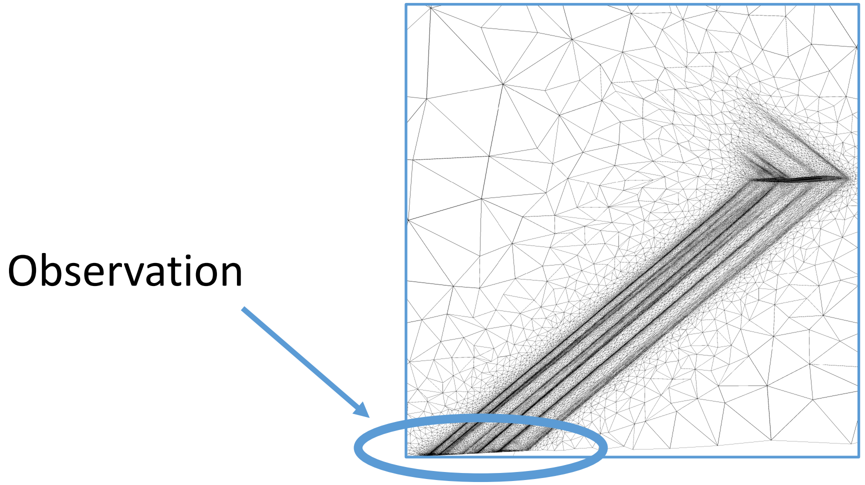
$$\|W - \Pi_h W\|_{L^p(\Omega_h)}$$

Goal-oriented error estimates

- Derive the ’best’ mesh to observe a given functional

$$j(w) = (g, w)$$

- Optimal control of the interpolation error in L^p norm:

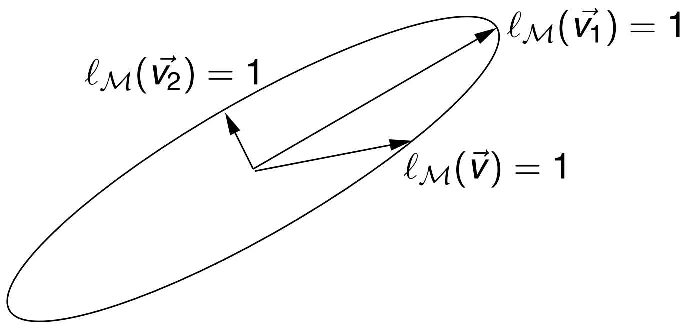


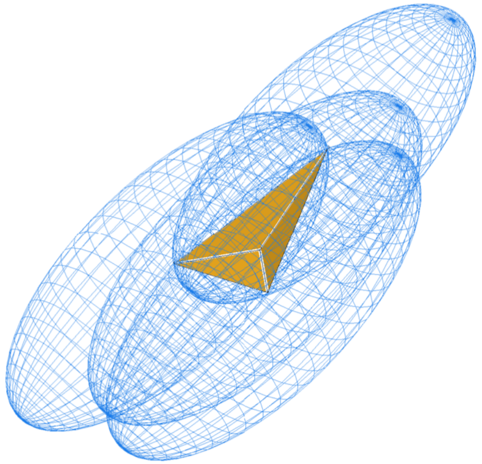
$$|J(W) - J(W_h)| \approx \left\|\nabla W^* \cdot \left(\mathcal{F}(W) - \Pi_h \mathcal{F}(W)\right)\right\|_{L^2(\Omega_h)}$$

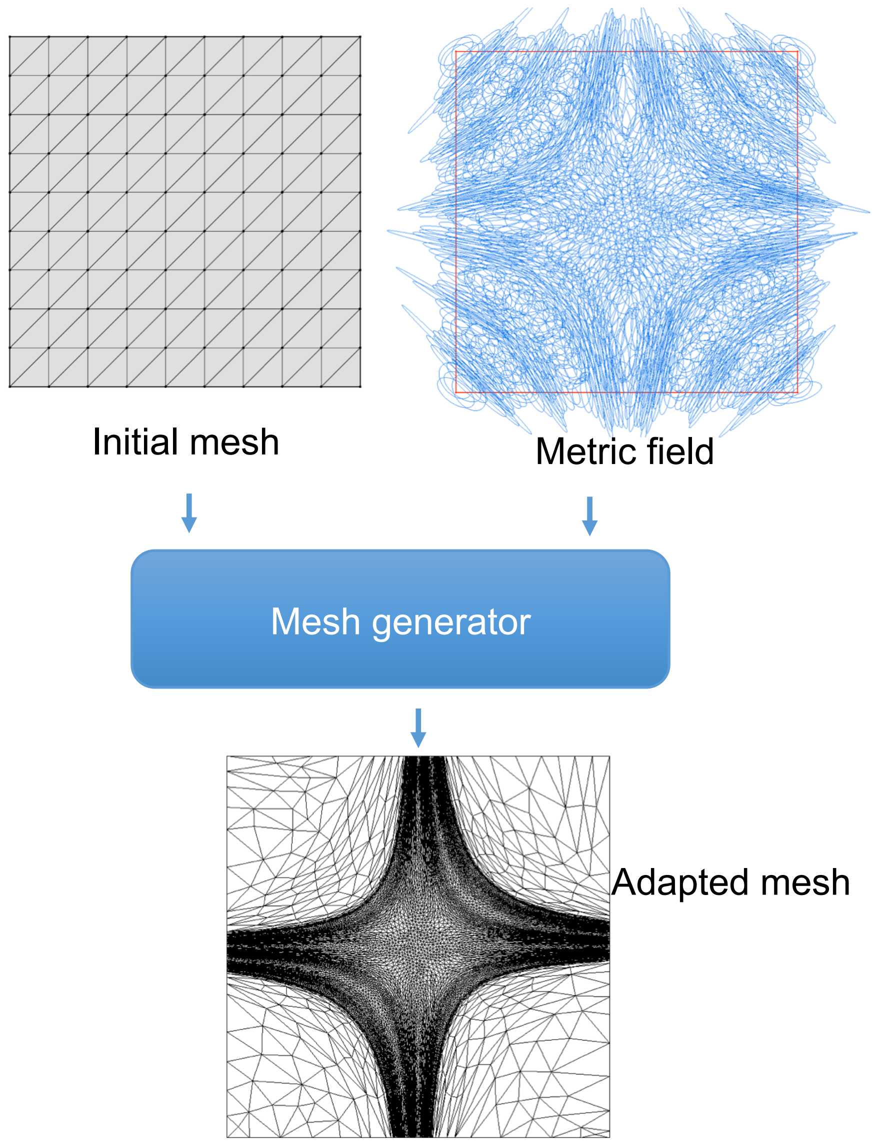
Anisotropic mesh generation

- Generate a unit mesh w.r.t. the input metric field.
- Local mesh operations are performed:
- Standard mesh operators: point insertion/deletion, edge swap/collapse, etc.
- In AMG: all these operations are embedded in

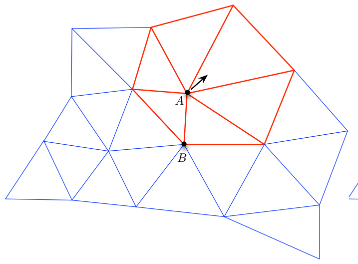
one cavity-based operator



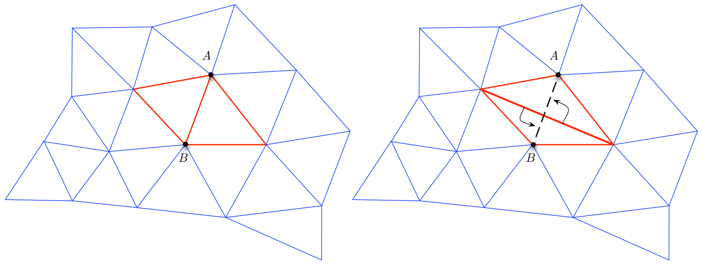




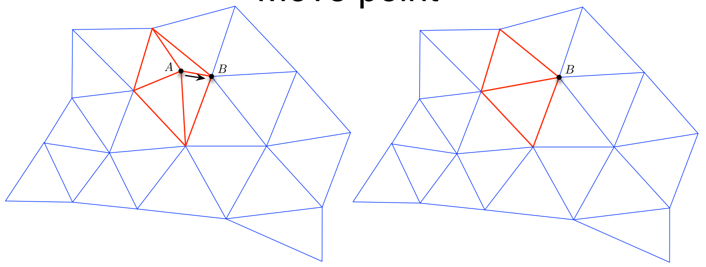
Standard meshing operators



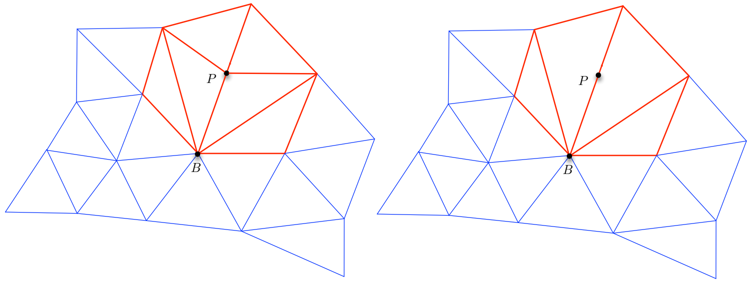
Move point



Swap edge



Collapse edge



Point insertion

- Step 1 : Generate a unit mesh
  - Split long edges in the metric
  - Collapse small edges in the metric
- Step 2 : Optimization
  - Perform point smoothing and swaps

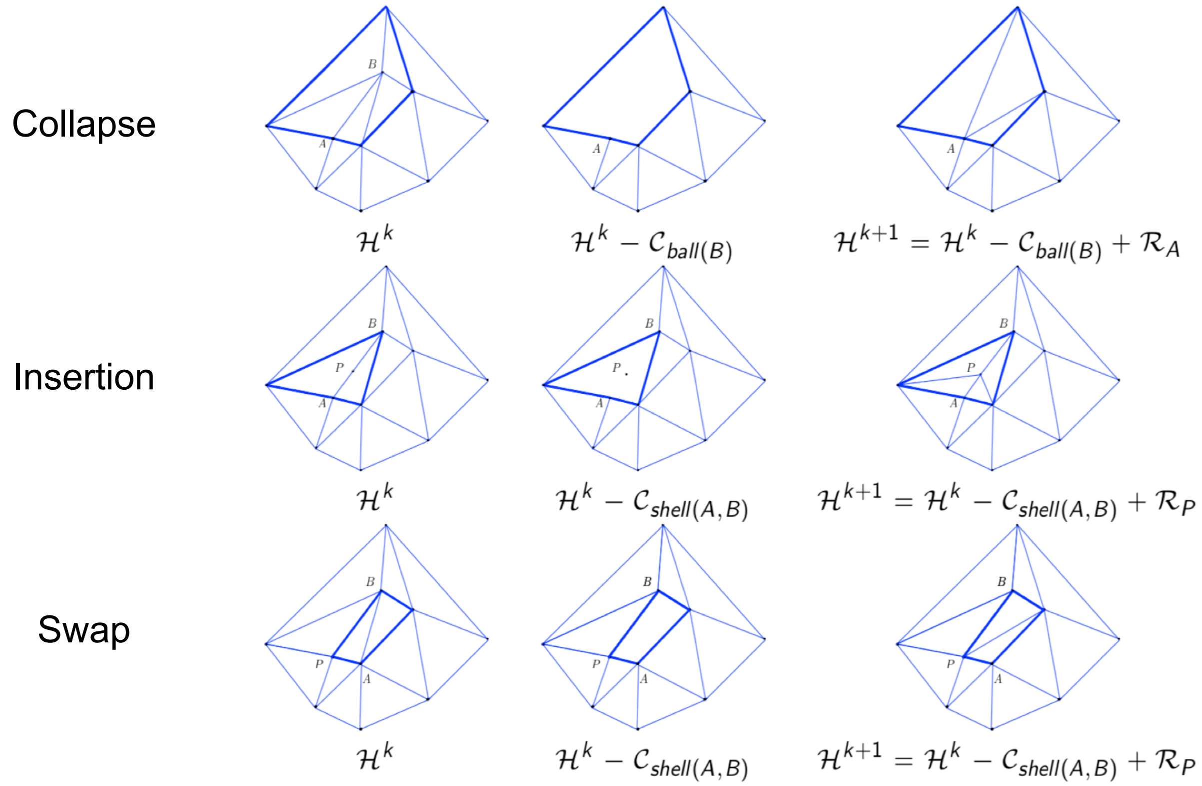
- All these operators are applied sequentially and are monitored by a quality function

Unique cavity-based operator

[Loseille and Löhner, AIAA 2010, Loseille et Menier, IMR 2013]

▸ In PyAMG:

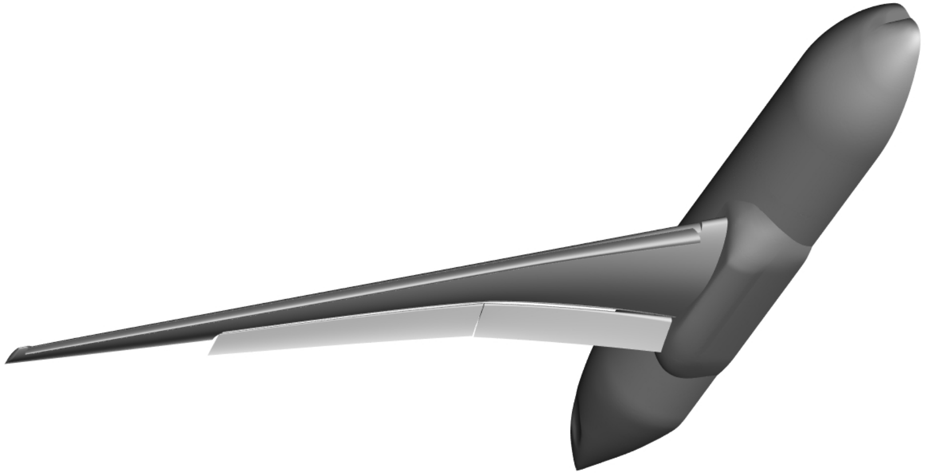
- Each mesh operation corresponds to a node (re)insertion



- Unified : all-in-one
- Generalized : multiple-operators at once
  - Do either line, surface or volume
  - Handle non-manifold geometries
  - Easy to maintain and update
  - Uniform speed for all operators

HL-CRM

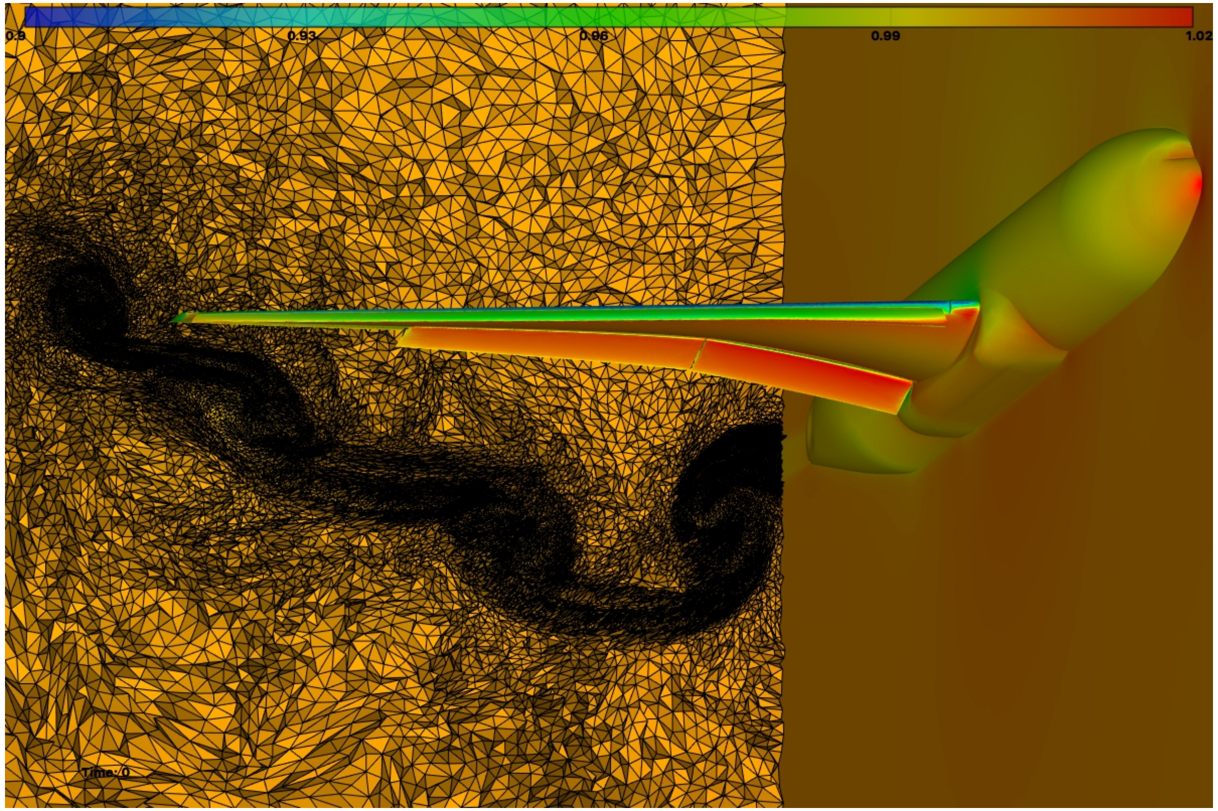
- Mach 0.2, 16 degrees, 3,2 Million
- Meshing process : start from a coarse mesh
- Geometry : P3 curved mesh (built from CAD)

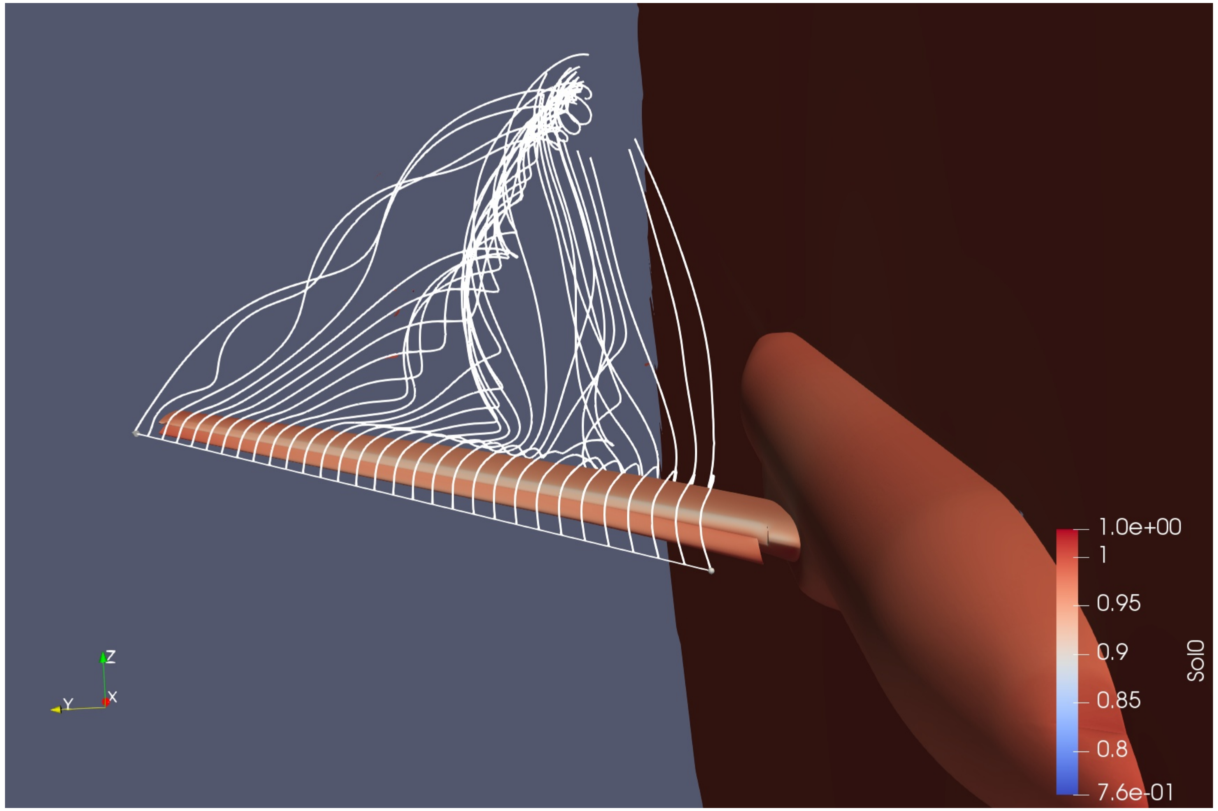


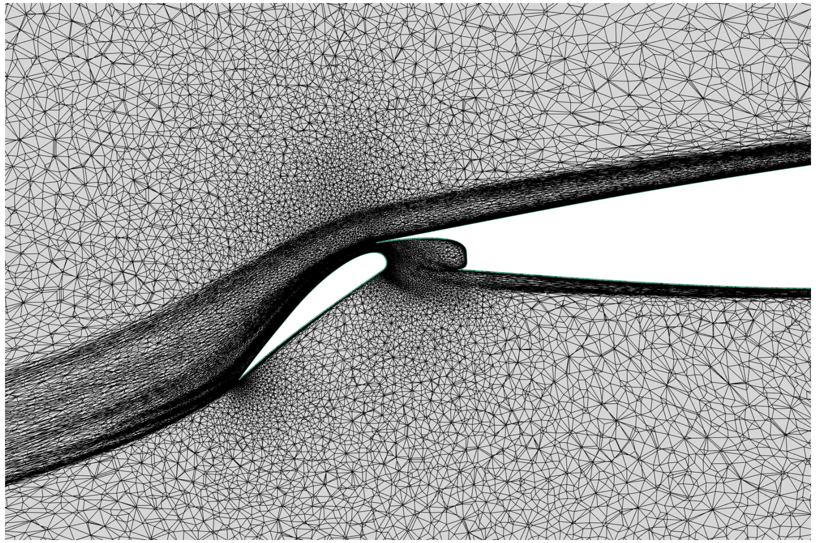
Software packages

- Wolf flow solver : FV/FE, HLLC scheme, Implicit, SA-Neg
- Goal oriented RANS error estimates : lift functional
- Fully anisotropic : no quasi-structured boundary layer mesh inserted

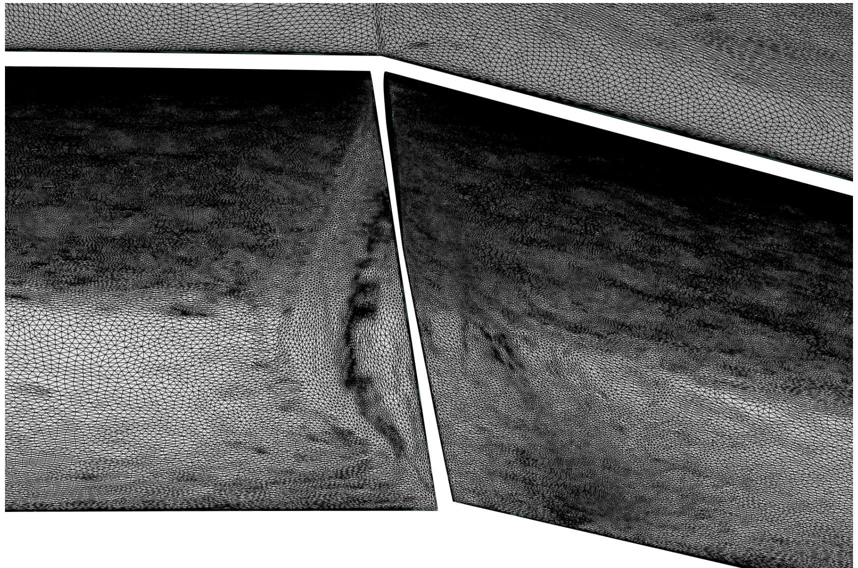
[L. Frazza Phd Thesis, 2018], [Alauzet and Frazza, Eccomas 2018], [Michal et al., AIAA 2018]

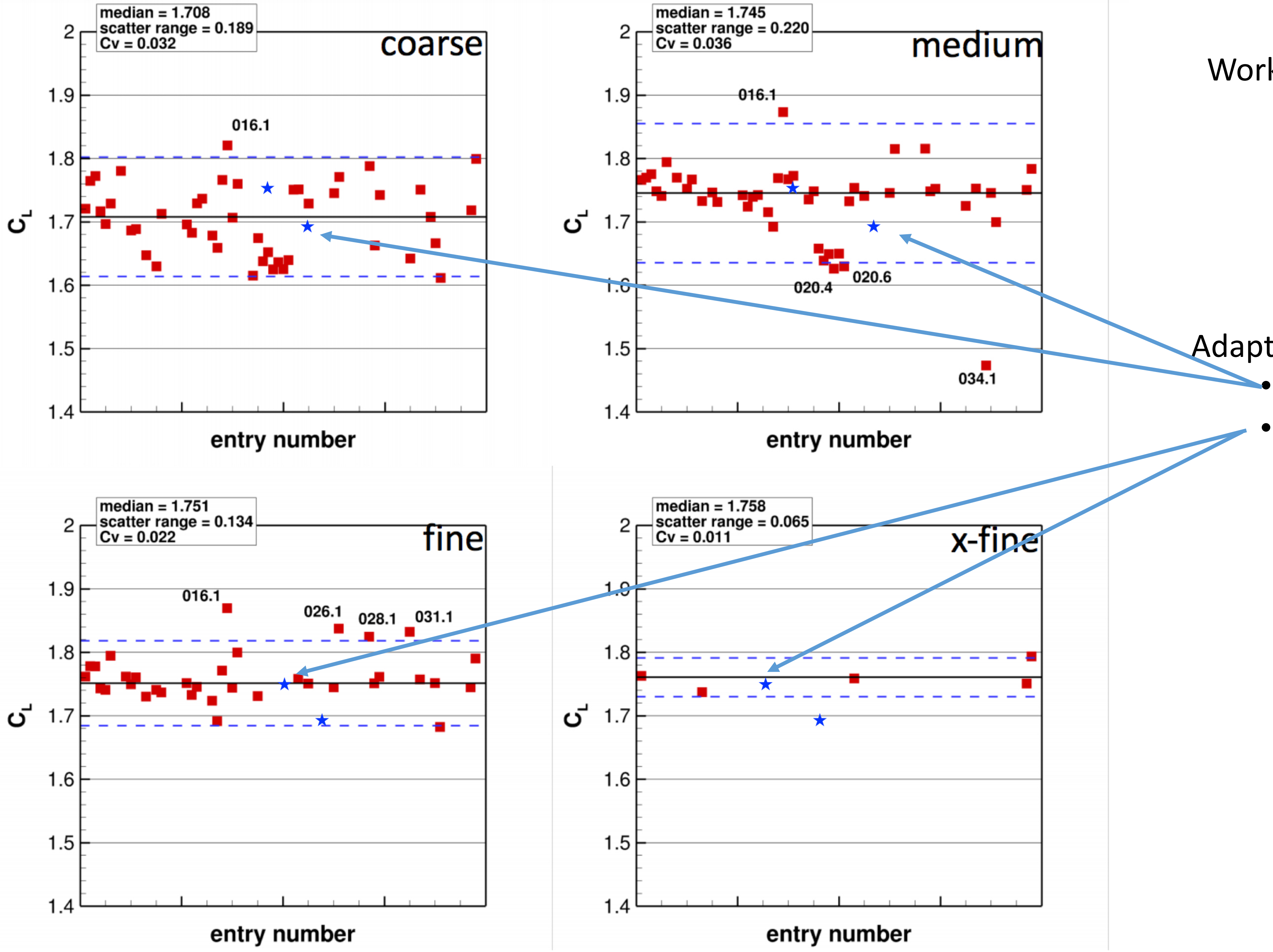






- Final mesh : 13 M vertices
- 10 Adaptations





Workshop grids :

- Coarse : 8 Million vertices
- Medium : 26 Million vertices
- Fine : 70 Million vertices
- X-Fine: 206 Million vertices

Adaptive grids : 10 adaptations

- Level 1 : 3,1 Million vertices
- Level 2 : 13 Million vertices

Outline

- Background on anisotropic mesh adaptation
  - Metric-based mesh generation
  - Error estimates
  - Unique cavity-based operators
  - Numerical examples : RANS Adaptation
- PyAMG interface and SU2
  - Python bindings
  - Numerical adaptive examples

Challenges of exposing adaptivity

- Automaticity : Minimal user intervention/parameters
- Geometry : surface approximation/projection/CAD
- Mesh Quality/Anisotropy ⟷ Solver Stability/Accuracy
- Error estimates : theoretical developments are still on-going

Typical issues with CFD solvers

- Associativity of boundary conditions to initial Geometry/CAD are lost ..
- Restart capabilities
- Strong solvers : stability and convergence on highly anisotropic elements

PyAMG : technical details

- Use of Python : Mesh, Solution and Options are dictionaries

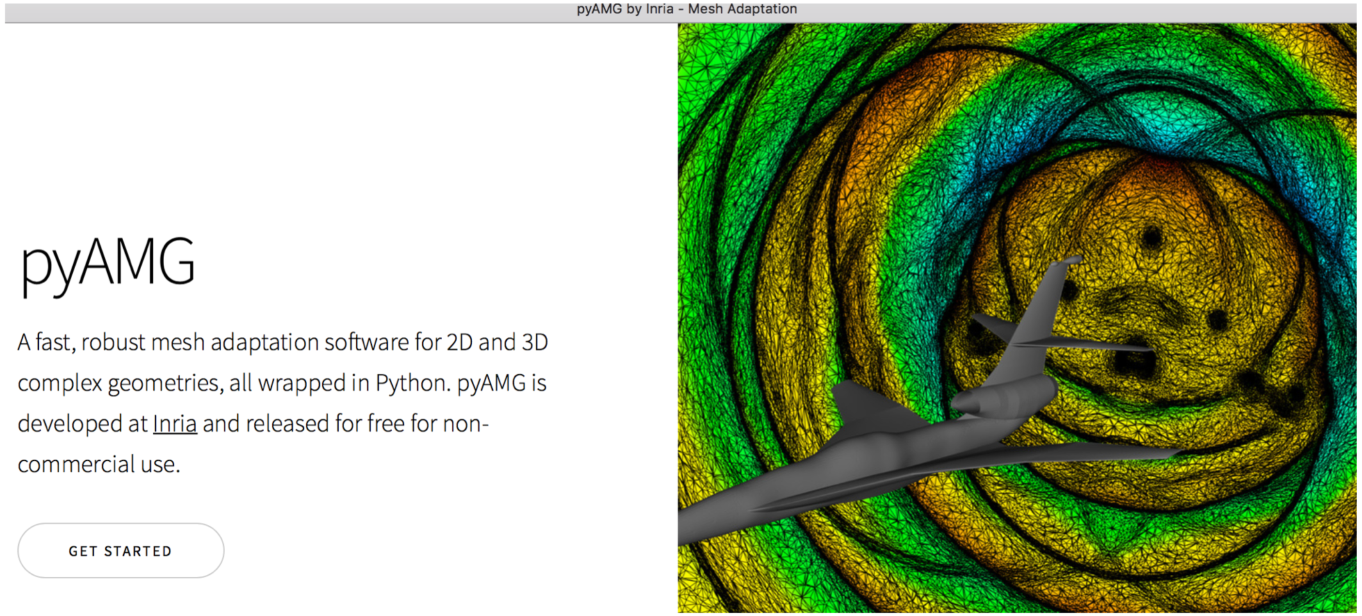
- Expose first a reduced set of capabilities
  - P3 High-order geometry — *prior to expose* CAD projection
  - Multi-scale error estimate — *prior to expose* Goal-oriented
  - Fully anisotropic — *prior to expose* adaptive quasi-structured BL

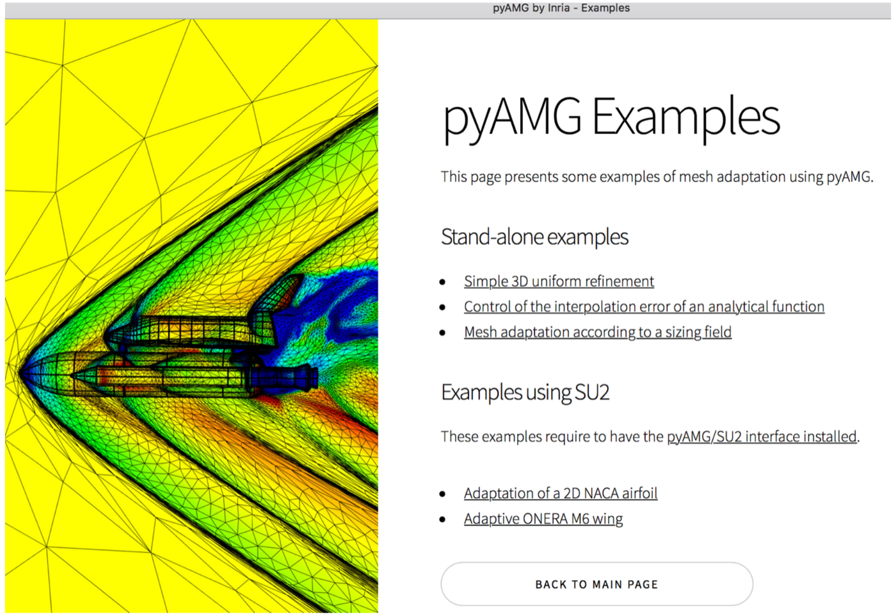
- Have a reduced number of user-parameters for mesh adaptation
  - Accuracy level
  - Gradation of the mesh
  - Sensor

- Full integration with SU2 in config file

PyAMG website

- https://pyamg.saclay.inria.fr





PyAMG : example of mesh definition

```
import pyamg
mesh = {}
mesh['xy']          = [ [0,0], [1,0], [1,1], [0,1] ]
mesh['Triangles'] = [ [1,2,3,0],[3,4,1,0]]
print "Writing initial_mesh.mesh"
pyamg.write_mesh(mesh, "initial_mesh.mesh")
remesh_options = {'hmax':0.5}
print "Refining mesh and writing refined_mesh.mesh"
adapted_mesh = pyamg.adapt_mesh(mesh,remesh_options)
pyamg.write_mesh(adapted_mesh, "refined_mesh.mesh")
```

- Mesh-related Keywords include : ‘xy’, ‘xyz’, ‘Triangles’, ‘Tetrahedra’, ’Edges’, ’Corners’
- Solutions is stored in key ‘sensor’
- Option-related keywords include : ’hmax’, ’hmin’, ‘Lp’, ‘gradation’

PyAMG : Example of mesh adaptation

```
# Define remeshing options
remesh_options              = {}
remesh_options['Lp']        = 2
remesh_options['gradation'] = 1.5
remesh_options['target']    = 20000

# Adapt the mesh, perform 5 iterations
print "   Anisotropic adaptation : iteration 1"
msh3d_aniso = pyamg.adapt_mesh(msh3d, remesh_options)

for ite in range(2,5):
  print "   Anisotropic adaptation : iteration %d " %(ite)
  msh3d_aniso['sensor'] = create_sensor(msh3d_aniso)
  msh3d_aniso           = pyamg.adapt_mesh(msh3d_aniso, remesh_options)

# Output final mesh
pyamg.write_mesh(msh3d_aniso,"cube_aniso.meshb")
```

PyAMG-SU2 : Example

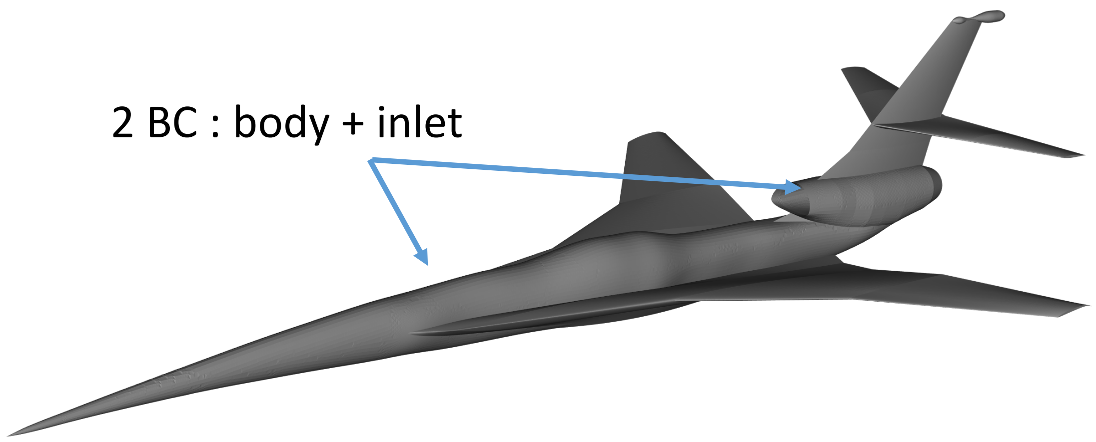
```
% ------------- MESH ADAPTATION PARAMETER ------------%

% Mesh size parameters
ADAP_SIZES= (2000, 4000, 8000)

% Number of iterative loops performed for each prescribed size
ADAP_SUBITE= (2, 2, 2)

% Number of CFD iterations for each mesh size
ADAP_EXT_ITER= (1000, 1000, 1000)

% Prescribed residual reduction for each mesh size
ADAP_RESIDUAL_REDUCTION= (3, 3, 3)

% Sensor used for mesh adaptation (MACH, PRES, or MACH_PRES)
ADAP_SENSOR=  MACH

% Max and min edge sizes
ADAP_HMAX=  200
ADAP_HMIN=  1e-8

% Prescribed mesh gradation
ADAP_HGRAD= 1.3

% Output adapted mesh
MESH_OUT_FILENAME= M6_adap.su2

% Final adapted restart solution
RESTART_FLOW_FILENAME= M6_adap.dat
```

- Everything is contained in the config file
- PyAMG bindings are transparent to the user

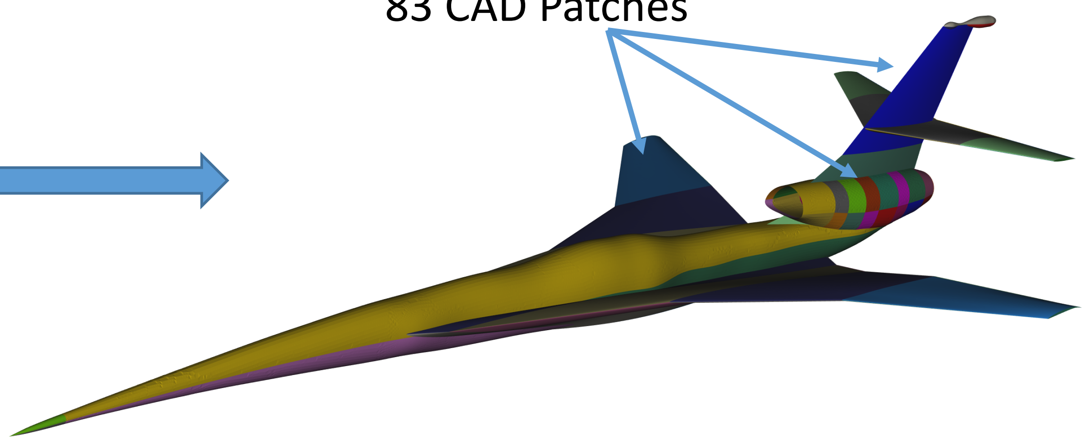
```
$ python mesh_adaptation_amg.py -f adap_ONERAM6.cfg
```

PyAMG-SU2 : Behind the scene

- Convert SU2 data structures into Python dictionaries
- Expand/Split MARKERS to be as close as possible of the most detailed geometry description (CAD Patch)



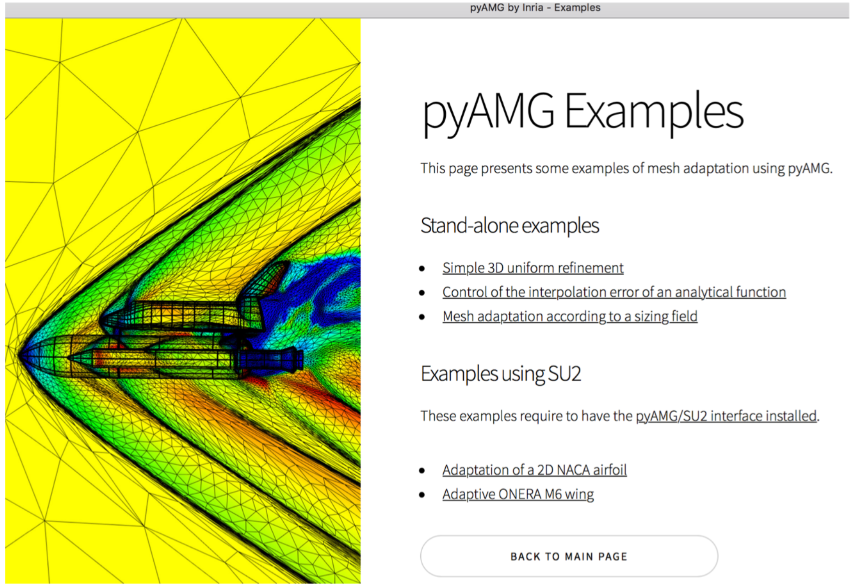
CFD Boundary condition



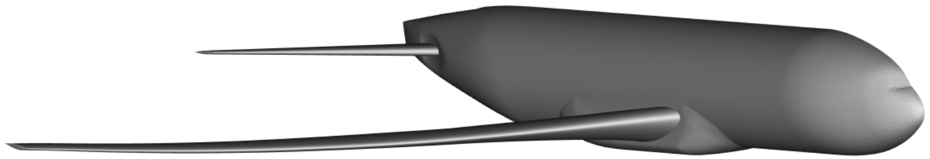
Good practice

PyAMG-SU2 : current status

- Inviscid and RANS with frozen boundary layers are support
- Support to fully unstructured boundary layers is on-going
- The adaptive script is still a custom branch of SU2



PyAMG-SU2 : Inviscid CRM

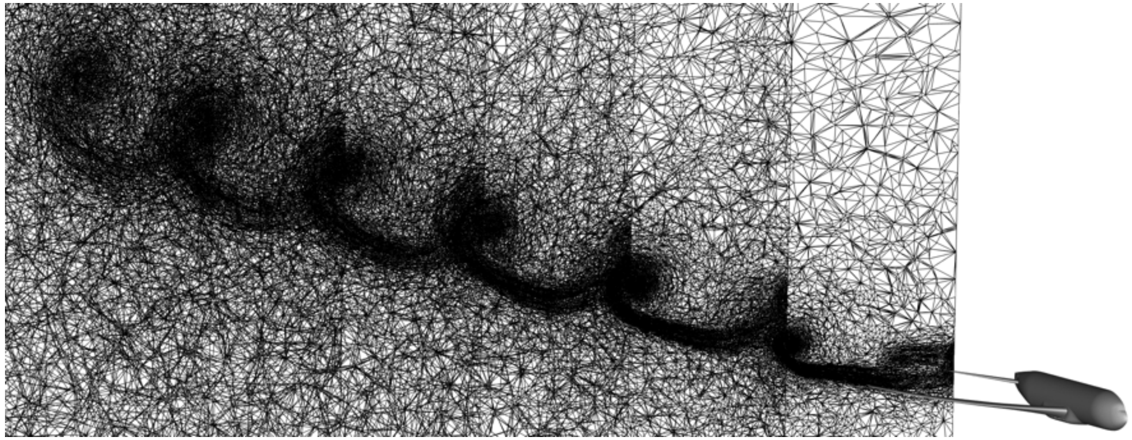


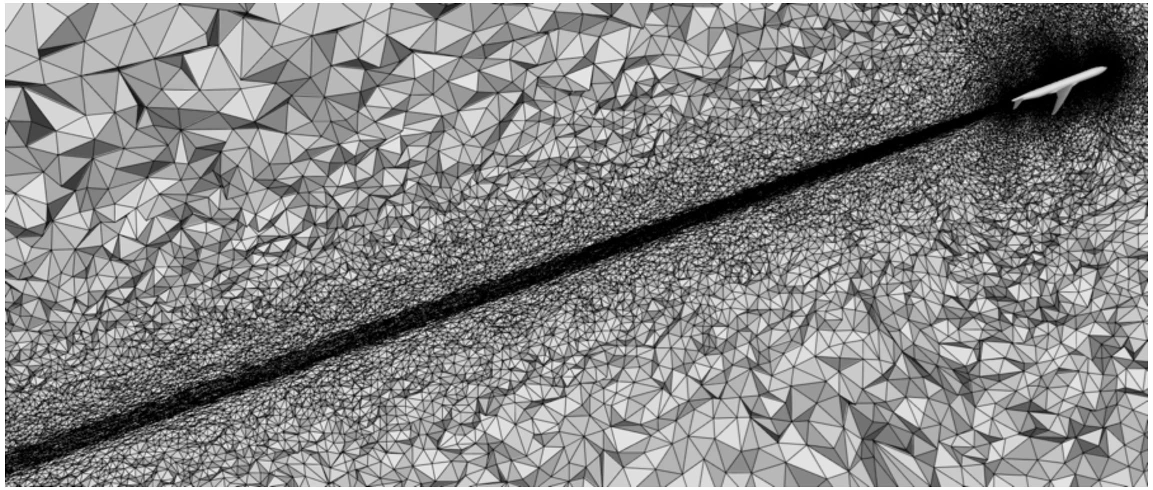
- Mach 0.85, 2.133 degrees, Inviscid
- Meshing process : start from a coarse mesh
- Geometry : P3 curved mesh (built from CAD)

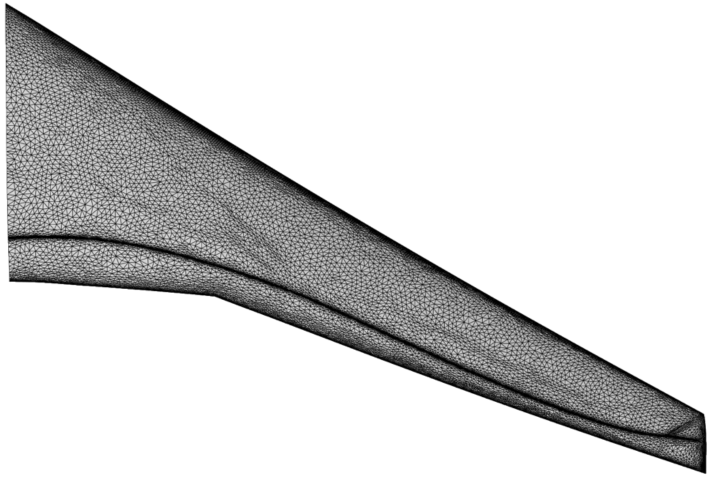
Software packages

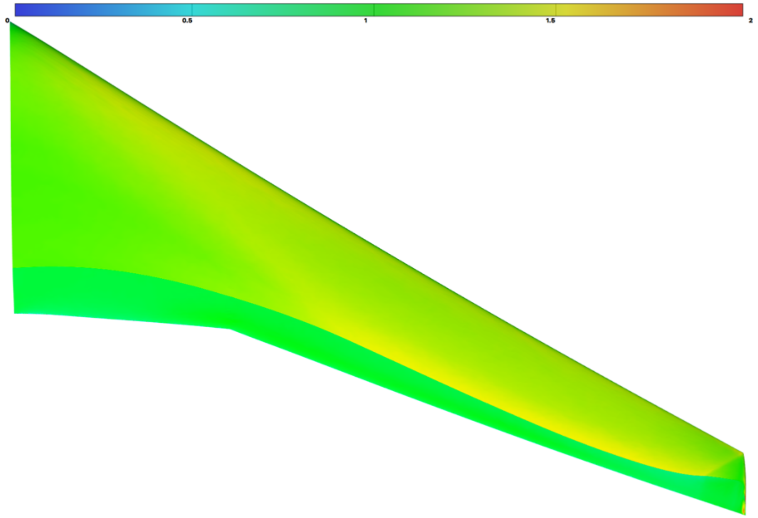
- SU2 : JST, MG, FMGRES
- Multi-scale error estimate : Mach Number
- Final mesh 3 019 832 vertices

PyAMG-SU2: Inviscid CRM

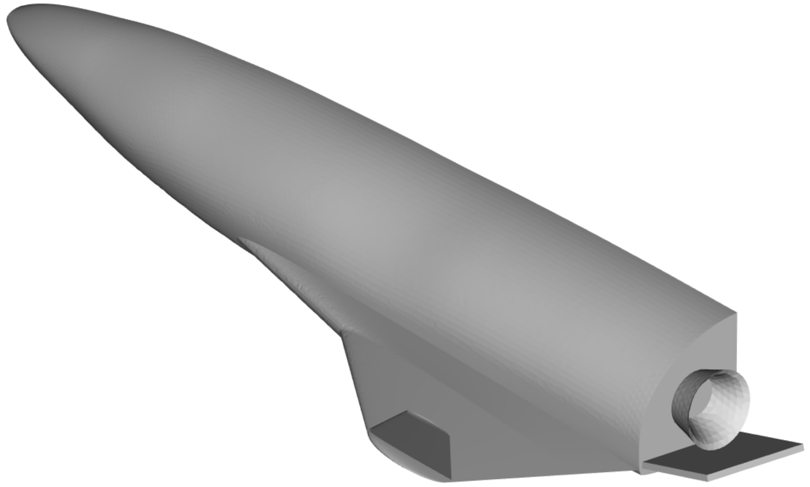








PyAMG-SU2 : RANS Case

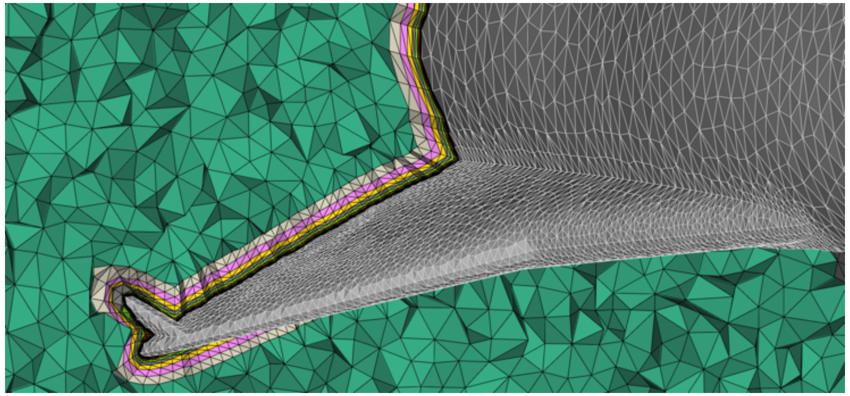


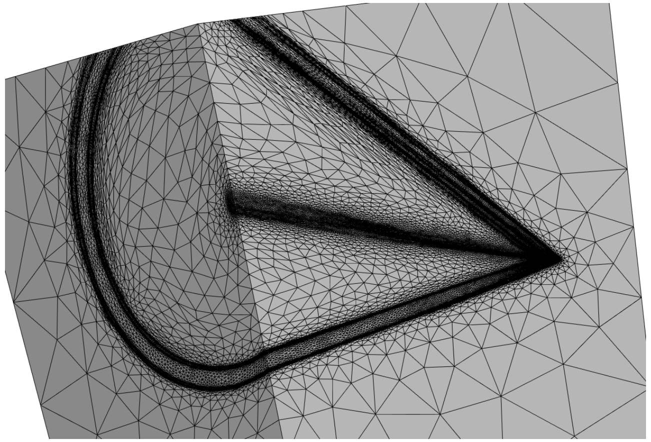
- Mach 2.0, 10 degrees, 1.65 Million
- Meshing process : start from a coarse mesh
- Geometry : P3 curved mesh (built from CAD)

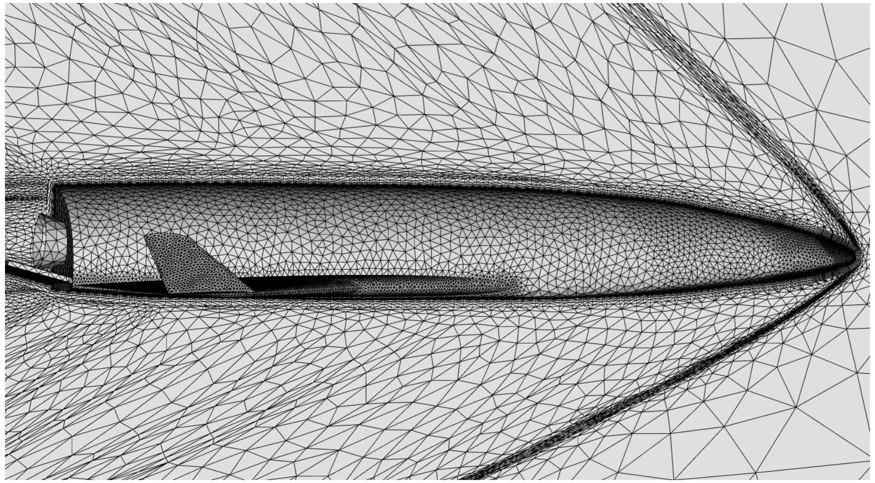
Software packages

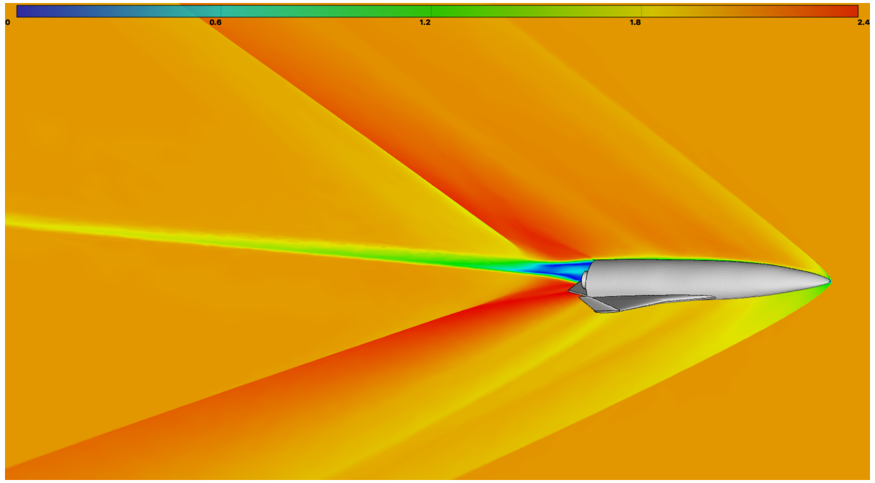
- SU2 : SA, JST, FMGRES
- Multi-scale error estimate : Mach Number
- Frozen boundary layer
- Final mesh 1 343 684 vertices (outside BL)

PyAMG/SU2: RANS case









Conclusion

- Beta version of adaptive mesh generation in 2D/3D
- Error estimates, interpolation, surface, volume mesh generations
- Inviscid runs, RANS adaptations

On-going work

- Increase the set of documented adaptive test cases in 2D/3D
- Detail best practice to start a new adaptive computation (Geometry)

Stay updated

- Visit pyamg.saclay.inria.fr for more details
- Mailing list : send an email to pyamg-request@inria.fr with subscribe as topic

Acknowledgements

- Frédéric Alauzet and Loïc Frazza :

  Wolf flow solver, Adjoint RANS error estimates
- Julien Vanharen and Mathieu Rigal :

  Python bindings (PyMeshb, PyViz)
- Unstructured Grid Adaptation Working Group

  https://github.com/UGAWG
- Visualization software : ViZiR team (INRIA)

  https://vizir.inria.fr/

Thank you for your attention !

pyamg.saclay.inria.fr

pyAMG

A fast, robust mesh adaptation software for 2D and 3D complex geometries, all wrapped in Python. pyAMG is developed at Inria and released for free for non-commercial use.

GET STARTED